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THE PROPOSITIONS

IN

MECHANICS AND HYDROSTATICS,

WHICH ARE REQUIRED OF QUESTIONISTS,
NOT CANDIDATES FOR HONORS,

WITH

ILLUSTRATIONS AND EXAMPLES,

COLLECTED FROM VARIOUS SOURCES.

By A. C. BARRETT, M.A.

CAMBRIDGE:
J. & J. J. DEIGHTON;
LONDON: JOHN W. PARKER.

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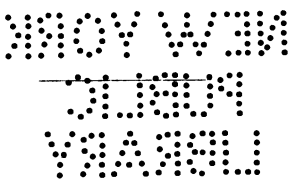
PREFACE.

THE new regulation with respect to the mathematical subjects required in the ordinary Examination for Degrees, appears to demand a work of a nature somewhat different from those which have hitherto been in use at this University. To supply this deficiency is the object of the present publication. The proofs of the propositions will be found to differ but little from those given in other works upon the subject; in one or two instances the author has departed from the arrangement in the Schedule, where it appeared to him that by so doing the connexion between the different propositions would be rendered more clear. In the proofs of the propositions, as well as in the explanations of the experiments described in the notes, he has aimed at perspicuity rather than brevity, being fully convinced that the reader who thoroughly understands the principle of a demonstration, will have little difficulty in condensing it for himself; although a demonstration, however brief, will seldom be written out correctly if it be not clearly understood.

In a treatise like the present, an extended view of the subject would be impracticable; all that has been

attempted in the following pages is to render the reader able to explain for himself the simpler physical phenomena connected with the subject, which are of frequent occurrence in everyday life; and to understand the common forms of those instruments, which are absolutely necessary in the daily occupations of mankind.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SCHEDULE

OF THE

PROPOSITIONS IN MECHANICS AND HYDROSTATICS,

WHICH ARE REQUIRED OF

QUESTIONISTS, NOT CANDIDATES FOR HONORS.

MECHANICS.

DEFINITIONS of Force, Weight, Quantity of Matter, Density, Measure of Force.

THE LEVER.

Definition of the Lever.

Axioms.

Prop. 1. A horizontal prism or cylinder of uniform density will produce the same effect by its weight as if it were collected at its middle point.

Prop. 2. If two weights acting perpendicularly on a straight lever on opposite sides of the fulcrum balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to their sum.

Prop. 3. If two forces acting perpendicularly on a straight lever in opposite directions and on the same side of the fulcrum balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to the difference of the forces.

Prop. 4. To explain the different kinds of levers.

Prop. 5. If two forces acting perpendicularly at the extremities of the arms of any lever balance each other, they are inversely as the arms.

Prop. 6. If two forces acting at any angles on the arms of any lever balance each other, they are inversely as the perpendiculars drawn from the fulcrum to the directions in which the forces act.

Prop. 7. If two weights balance each other on a straight lever when it is horizontal, they will balance each other in every position of the lever.

COMPOSITION AND RESOLUTION OF FORCES.

Definition of Component and Resultant Forces.

Prop. 8. If the adjacent sides of a parallelogram represent the component forces in direction and magnitude, the diagonal will represent the resultant force in direction and magnitude.

Prop. 9. If three forces, represented in magnitude and direction by the sides of a triangle, act on a point, they will keep it at rest.

Definition of Wheel and Axle.

Prop. 10. There is an equilibrium upon the wheel and axle when the power is to the weight as the radius of the axle to the radius of the wheel.

Definition of Pulley.

Prop. 11. In the single moveable pulley where the strings are parallel, there is an equilibrium when the power is to the weight as 1 to 2.

Prop. 12. In a system in which the same string passes round any number of pullies and the parts of it between the pullies are parallel, there is an equilibrium when Power (P) : Weight (W) :: 1 : the number of strings at the lower block.

Prop. 13. In a system in which each pulley hangs by a separate string and the strings are parallel, there is an equilibrium when $P : W :: 1$: that power of 2 whose index is the number of moveable pullies.

Prop. 14. The weight (W) being on an inclined plane and the force (P) acting parallel to the plane, there is an equilibrium when $P : W$ :: the height of the plane : its length.

Definition of Velocity.

Prop. 15. Assuming that the arcs which subtend equal angles at the centers of two circles are as the radii of the circles, to shew that if P and W balance each other on the wheel and axle, and the whole be put in motion, $P : W ::$ W's velocity : P's velocity.

Prop. 16. To shew that if P and W balance each other in the machines described in Propositions 11, 12, 13, and 14, and the whole be put in motion, $P : W ::$ W's velocity in the direction of gravity : P's velocity.

THE CENTER OF GRAVITY.

Definition of Center of Gravity.

Prop. 17. If a body balance itself on a line in all positions, the center of gravity is in that line.

Prop. 18. To find the center of gravity of two heavy points; and to shew that the pressure at the center of gravity is equal to the sum of the weights in all positions.

Prop. 19. To find the center of gravity of any number of heavy points; and to shew that the pressure at the center of gravity is equal to the sum of the weights in all positions.

Prop. 20. To find the center of gravity of a straight line.

Prop. 21. To find the center of gravity of a triangle.

Prop. 22. When a body is placed on a horizontal plane, it will stand or fall, according as the vertical line, drawn from its center of gravity, falls within or without its base.

Prop. 23. When a body is suspended from a point, it will rest with its center of gravity in the vertical line passing through the point of suspension.

HYDROSTATICS.

Definitions of Fluid ; of elastic and non-elastic Fluids.

PRESSURE OF NON-ELASTIC FLUIDS.

Prop. 1. Fluids press equally in all directions.

Prop. 2. The pressure upon any particle of a fluid of uniform density is proportional to its depth below the surface of the fluid.

Prop. 3. The surface of every fluid at rest is horizontal.

Prop. 4. If a vessel, the bottom of which is horizontal and the sides vertical, be filled with fluid, the pressure upon the bottom will be equal to the weight of the fluid.

Prop. 5. To explain the *hydrostatic paradox*.

Prop. 6. If a body floats on a fluid, it displaces as much of the fluid as is equal in weight to the weight of the body ; and it presses downwards and is pressed upwards with a force equal to the weight of the fluid displaced.

SPECIFIC GRAVITIES.

Definition of Specific Gravity.

Prop. 7. If M be the magnitude of a body, S its specific gravity, and W its weight, $W = MS$.

Prop. 8. When a body of uniform density floats on a fluid, the part immersed : the whole body :: the specific gravity of the body : the specific gravity of the fluid.

Prop. 9. When a body is immersed in a fluid, the weight lost : whole weight of the body :: the specific gravity of the fluid : the specific gravity of the body.

Prop. 10. To describe the *hydrostatic balance*, and to shew how to find the specific gravity of a body by means of it, 1st, when its specific gravity is greater than that of the fluid in which it is weighed ; 2ndly, when it is less.

Prop. 11. To describe the common *hydrometer* and to shew how to compare the specific gravities of two fluids by means of it.

ELASTIC FLUIDS.

Prop. 12. Air has weight.

Prop. 13. The elastic force of air at a given temperature varies as the density.

Prop. 14. The elastic force of air is increased by an increase of temperature.

Prop. 15. To describe the construction of the common *air-pump* and its operation.

Prop. 16. To describe the construction of the *condenser* and its operation.

Prop. 17. To explain the construction of the common *barometer*, and to shew that the mercury is sustained in it by the pressure of the air on the surface of the mercury in the basin.

Prop. 18. The pressure of the atmosphere is accurately measured by the weight of the column of mercury in the barometer.

Prop. 19. To describe the construction of the common *pump* and its operation.

Prop. 20. To describe the construction of the *forcing-pump* and its operation.

Prop. 21. To explain the action of the *siphon*.

Prop. 22. To shew how to graduate a common *thermometer*.

Prop. 23. Having given the number of degrees on *Fahrenheit's* thermometer, to find the corresponding number on the *Centigrade* thermometer.

By a GRACE of the Senate, passed in 1837, it was decided, (Clause 9) "That the papers in the Mathematical Subjects shall consist of Propositions in Mechanics and Hydrostatics, according to the annexed Schedule," (to which a subsequent Grace, passed in March, 1846, has added the following clause) "and also, of such Questions and Applications as arise directly out of the afore-mentioned Propositions."

ERRATA.

PAGE	LINE		FOR		READ
11	21		placed at N		placed at M and N
12	27		produce Q		balance Q
14	9		fender		grate
15	19 and 23		Q		Q'
19	2	*after*	same point	*insert*	but not in the same right line
24	in fig. and Cor. 1.		Q		W
...	15		(III. 17)		(III. 18)
26	9 and 28		same direction as the weight		opposite direction to that in which the weight acts
43	12		increased or diminished		diminished or increased
44	8		other portion		other equal portion
46	17		(I. 34)		(I. 33)
54	in margin		w and w'		W and W'
55	2		W'		w'
...	19		$\frac{W}{w+W'-w'}$		$\frac{w}{w+W'-w'}$
56	24		40 divisions		30 divisions
57	10		increased or diminished		diminished or increased
58	24		through C		through A
59	3		we find that		we find by experiment that
...	6		fF		fF'
63	16		A		it
65	22		closes f		keeps f closed
67	6		shuts f		keeps f closed
...	11	*after*	the barrel B	*insert*	when f will shut by its own weight
69	13 and 16		on		in
...	18		Ba, Bd		ba, bd
...	20 and 21		a		A
70	1	*after*	the bulb	*insert*	and part of the tube

NOTES.

78	6		colour		caloric
84	Ex. 13		pounds		cubic inches
87	6		of the fulcrum		on the fulcrum
88	19		dA		dB
...	...		Ac		Ae
91	2		weight		weights
102	Ex. 22		2 cwt.		1 cwt.
119	22		$\frac{P\text{'s velocity, \&c.}}{W\text{'s velocity, \&c.}}$		$\frac{W\text{'s velocity, \&c.}}{P\text{'s velocity, \&c.}}$
120	21		ditto		ditto
...	24		P's velocity, &c. : W's velocity, &c.		W's velocity, &c. : P's velocity, &c.
121	...		(v. 15)		(I. 15)
141	Ex. 12		angle		triangle
158	Ex. 3	*after*	S = S. G. of fluid	*insert*	M = its magnitude or volume
...	...		$S \times \frac{3}{5} =$ &c. $= 1 \times .6$		$S \times \frac{3M}{5} = M \times .6$
174	27	*after*	weather	*insert*	is
189	Int. *Ch. Ex.* 14		7.8725		7.875
190	*Bodies immersed in fluids, Ex.* 32		$92\frac{9}{10}$ oz.		$92\frac{2}{19}$ inches.

MECHANICS.

CHAPTER I.

DEFINITIONS AND PRINCIPLES.

SECTION I.

I. MECHANICS is the science which treats of the laws of the rest and motion of bodies.

That branch of the science, which treats of the laws of the rest of bodies, is called Statics.

That branch of the science, which treats of the laws of the motion of bodies, is called Dynamics.

(In the following pages we shall treat only of the Elements of Statics.)

II. Whatever produces, or tends to produce any alteration whatever in the state of rest or motion of a body, is called a force.

III. All bodies if left to themselves would fall to the earth's surface, and this tendency which bodies have to fall to the earth, is called gravity.

IV. The unknown cause by which this tendency is produced, is called gravitation.

V. The precise amount of pressure, which any body at rest exerts in the direction of the earth, is called the weight of that body.

[illegible] distance
[illegible] extension
[illegible] essentially be-
[illegible] properties.
[illegible] mechanics.
[illegible] weight may be
[illegible] may define
[illegible] weight.

[illegible] composed
[illegible] quantity of matter, or quantity of weight.
[illegible] mass of a body.
[illegible] obtained
[illegible] possesses the same
[illegible] matter in different bodies are
[illegible] weights.

[illegible] denotes the closeness with which
[illegible] united.

[illegible] a certain body must
[illegible] the closeness with which
[illegible] united and may therefore
[illegible] the quantity of matter in a unit of
[illegible]

[illegible] may be taken as the unit of
[illegible] water will be the quantity of
[illegible] cubic inch of water.

[illegible] denote the density of the bodies
[illegible] supposed to be uniform, that is, the par-
[illegible] occupying any body are supposed to
[illegible] throughout the space occupied
[illegible]

The densities of different bodies are proportional to the weights of *any* equal volumes of those bodies.

For, in proportion as one body is denser than another, in the same proportion it must contain more heavy matter in the same volume than the other, and therefore, in the same proportion, a volume of the one will weigh more than an equal volume of the other.

XI. The specific gravity of any particular body, is the weight of a unit of volume of that body.

XII. Let S be the specific gravity of a body, D its density, W its weight, Q its mass, i.e. the quantity of matter contained in it, M its volume or magnitude, i.e. the number of cubic inches contained in it.

Q = number of cubic inches in the body × the quantity of matter in one cubic inch,

$= \ldots\ldots\ldots\ldots M \,.\, D$;

W = number of cubic inches in the body × the weight of one cubic inch,

$= \ldots\ldots\ldots\ldots M \,.\, S.$

XIII. If the matter in any body be compressed so that M becomes M', and D, D'.

Then if the quantity of matter remain the same,

$MD = Q$, and $M'D' = Q$; $\therefore MD = M'D'$;

and $M : M' :: D' : D$,

or, the magnitude varies inversely as the density.

SECTION II.

ON FORCES.

I. STATICAL forces are sometimes called pressures; and their magnitude is measured by the number of pounds, which they would support, if made to act directly opposite to the force of gravity.

II. The point at which a force acts upon a body is called the point of application of that force.

III. The line in which a force produces, or tends to produce motion, is called the line of the force's action.

IV. Not only this line, but also every line that is parallel to this line, is said to be in the direction of the force's action, or (more briefly) in the direction of the force.

V. In order to investigate the effects produced by a force, we must know:

1. The point of application of the force.
2. Its direction.
3. Its magnitude.

VI. If two or more forces be applied to a body, and no motion is produced, the body is said to be in equilibrio, and the forces are said to counteract each other.

VII. The effect produced by any force may be made to take place in any direction, by applying the force to a string, which passes over a fixed point or circular pulley; and it is found by experiment, that this effect is the same, whatever be the direction of the strings, provided we make allowance

for the weight of the strings, and the friction of the pulley.

VIII. We find also by experiment, that, if a force act by means of a string, and we make allowance for the weight of the string, the effect produced will be the same at whatever point of the string the force be applied, provided the direction in which the force acts be not changed.

Hence, the effect produced by any force is the same, at whatever point in its line of action it be applied.

IX. If a string be fastened at one end, and pulled by a force applied at the other, the resistance to motion made by the string at any point is called the tension of the string at that point. If the weight of the string be neglected, the tension of the string will be the same at every point, namely, the force with which the string is pulled.

X. Forces may properly be represented by right lines.

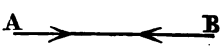
1. Since lines may be drawn from any point, lines may be drawn so as to represent the points of application of forces.

2. Since lines may be drawn in any direction, lines may be drawn so as to represent the directions of forces.

3. Since lines may be drawn of any length, lines may be drawn so as to bear to each other the same ratios that the forces do, and may therefore be drawn so as to represent the magnitudes of the forces.

Therefore lines may be drawn so as to represent forces in their points of application, directions, and magnitudes.

XI. A force acting along the line *AB*, from *A* to *B*, has not the same effect as an equal force acting along the line *AB* from *B* to *A*. On the contrary, two such forces would have exactly *opposite* effects, and may be distinguished by calling the first force *AB*, and the second *BA*.



CHAPTER II.

THE LEVER.

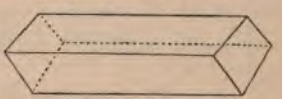
1. A PLANE, or plane superficies, is that surface, in which, if any two points be taken, the straight line which joins them lies wholly within the superficies.

A solid is that which hath length, breadth, and thickness.

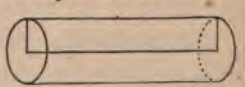
Planes are said to be parallel, when they do not meet, however far they may be produced.

A straight line is said to be perpendicular, or at right angles to a plane, when it makes right angles with every right line in that plane, which meets it.

2. A prism is a solid bounded by plane rectilinear figures, of which two are similar, equal, and parallel, and are called the ends of the prism; the other figures are all rectangles, and are called the sides of the prism.



A cylinder is the figure described by the revolution of a rectangle round one of its sides, which remains fixed. The fixed side is called the axis of the cylinder.



3. A lever is a rigid and inflexible rod moveable about one point in it. This point is called the fulcrum.

In the following propositions the lever is considered to be without weight, and to be moveable only in one plane.

4. The portions of the lever into which the fulcrum divides it are called the *arms* of the lever.

Axioms.

1. Equal forces acting perpendicularly at the extremities of equal arms of a lever, exert equal efforts to turn the lever round.

Hence, if they tend to turn the lever round in different directions, they balance each other.

2. If any two forces acting perpendicularly upon a horizontal straight lever, on opposite sides of a fulcrum, balance each other, the pressure on the fulcrum will be equal to the sum of the forces, and will act in the same direction as the forces.

3. If a single force, or two equal forces in the same direction, act perpendicularly on a straight lever, and be similarly situated with respect to two fulcrums on which the lever is supported, the pressure will be the same on either fulcrum.

4. And the pressure on the two fulcrums will be together equal to the whole force supported.

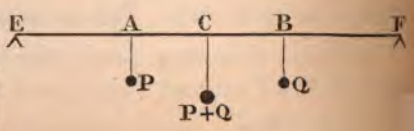
Prop. I.

A horizontal prism or cylinder of uniform density will produce the same effect by its weight as if it were collected at its middle point.

To prove this proposition we must first prove the following Lemma.

Any two equal weights acting perpendicularly on a horizontal straight lever, produce

If any two equal weights, P and Q act perpendicularly at the points A, B,



on the straight horizontal lever AF, moveable round the fulcrum F, they shall produce the same effect as if they were applied at C, the middle point between A and B.

the same effect as if collected at their middle point.

Produce BA to E, making $AE = BF$, and let a fulcrum be placed at E.

Then since P and Q are equal, and similarly situated with respect to E and F,

Pressure on fulcrum E = pressure on fulcrum F
$= \frac{1}{2}$ whole weight supported
$= \frac{1}{2}(P + Q)$. (by Ax. 3 and 4.)

Again, if P and Q were removed and a weight $P + Q$ were placed at C,

Pressure on fulcrum E = pressure on fulcrum F
$= \frac{1}{2}$ the weight supported
$= \frac{1}{2}(P + Q)$. (by Ax. 3 and 4.)

Therefore the pressure on E is the same in both cases, and the lever will be kept at rest in both cases, by the same force applied at E; that is, P and Q produce the same effect when they act at A and B that they do when they are collected at C.

Now we may suppose the cylinder to be divided into a number of small equal cylinders, by a number of equidistant planes parallel to the ends of the cylinder. These cylinders will consist of a number of pairs of equal weights, equidistant from the centre.

The cylinder may be supposed to be made up of a number of such pairs.

By the Lemma, every such pair produces the same effect as if it were collected at the centre.

Therefore the whole cylinder, being made up of a number of such pairs, will produce the same effect as if it were collected at the centre.

Therefore the cylinder produces the same effect as if it were collected at its centre.

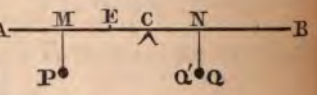
Cor. Hence such a cylinder would balance on a fulcrum placed at its middle point, and the pressure on the fulcrum would be the weight of the cylinder.

Prop. II.

If two weights acting perpendicularly on a straight lever, on opposite sides of the fulcrum, balance each other, they are inversely as their distances from the fulcrum, and the pressure on the fulcrum is equal to their sum.

Let the weights P, Q acting perpendicularly at M and N on the straight lever MN on opposite sides of the fulcrum C, balance each other.

Then $P : Q :: CN : CM$.



const. take Q' such that $P:Q'::CN:CM$ make $\begin{cases} NB=CM \\ MA=CN \\ ME=CN \end{cases}$

Take a weight Q' such that $P : Q' :: CN : CM$;

$\therefore$ componendo $P : P + Q' :: CN : CN + CM$, or MN,

take a cylinder AB such that its weight $= P + Q'$.

produce MN both ways, and make $NB = CM$, MA and ME each $= CN$; and let us suppose that there exists a cylinder, whose axis is AB, and whose density is uniform, and is such that its weight $= P + Q'$.

$CA = AM + MC = CN + NB = CB$; $\therefore$ C is the middle point of AB.

$AM \quad . \quad . \quad . \quad . \quad . \quad . \quad = ME$; $\therefore$ M is the middle point of AE.

$EN = EC + CN = EC + ME = CM = NB$; $\therefore$ N is the middle point of EB;

also $MN = CM + CN = CM + MA = CA = \frac{1}{2} . AB$.

CM and N are the middle points of AB, AE, EB.

Because C is the middle point of AB, the cylinder AB would balance round C; but the cylinder AB produces the same effect as its parts, AE, EB; and AE,

EB produce the same effect as if they were collected at their middle points M and N.

Therefore AE collected at M will balance EB collected at N.

AE collected at M balances EB collected at N.

$$\frac{\text{weight of } AE}{\text{weight of } AB} = \frac{AE}{AB}, \text{ since the density is uniform}$$

$$= \frac{\frac{1}{2}ME}{\frac{1}{2}MN} = \frac{CN}{MN} = \frac{P}{P+Q'},$$

but weight of $AB = P + Q'$; $\therefore$ weight of $AE = P$.

Weight of $AE = P$; weight of $EB = Q'$.

Hence weight of $EB = Q'$.

Therefore P at M and Q' at N will balance round C.

Therefore P and Q' balance.

But P at M balances Q at N, therefore Q and Q' produce the same effect and must therefore be equal.

Q produces the same effect as Q'; therefore $Q = Q'$, hence, $P:Q :: CN:CM$.

$$\text{Now } P : Q' :: CN : CM.$$

$$\therefore P : Q :: CN : CM. \qquad \text{Q.E.D.}$$

And the pressure on the fulcrum = the weight of the cylinder, (by Prop. 1.)

$$= P+Q', \text{ i.e.} = P+Q.$$

Q.E.D.

Cor. If two forces P, Q acting perpendicularly at M, N, on the straight lever MN, on opposite sides of the fulcrum C, are such that $P : Q :: CN : CM$.

Then P and Q will balance each other when placed at N.

For we have shewn in the proof of the last proposition, that if $P : Q' :: CN : CM$, P and Q' balance round C.

Cor. 2. Since $P : Q :: CN : CM$, by proportion, $P \times CM = Q \times CN$.

This formula will be found easier to recollect, and more convenient for working examples, than the proportion from which it is deduced.

Prop. III.

If two forces acting perpendicularly on a straight lever in opposite directions, and on the same side of the fulcrum balance each other,

(1) *The forces are inversely as their distances from the fulcrum, and* (2) *the pressure on the fulcrum is equal to their difference.*

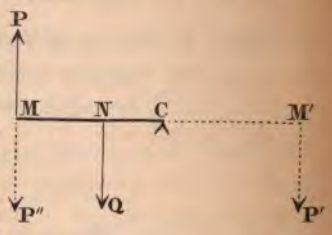
Let the two forces, P and Q, acting perpendicularly at M, N, on the straight lever CM, in opposite directions, and on the same side of the fulcrum C, balance each other.

Construction, take $CM' = CN$, P' and P'' each $= P$.

(1) $P : Q :: CN : CM$, and (2) the pressure on the fulcrum $= Q - P$.

In MC produced take $CM' = MC$, and let a force P' equal to P act at M' in the same direction as Q.

Then, since P and P' are equal forces acting perpendicularly on the lever at the end of equal arms CM, CM', and tending to turn the lever round in the same direction, by ax. 1, they produce the same effect, but P balances Q by hypothesis, therefore P' will produce Q.



P' balances Q; therefore, $P' : Q :: CN : CM'$; therefore, also, $P . Q :: CN : CM$.

Therefore, by the last proposition, $P' : Q :: CN : CM'$, but $P' = P$ and $CM = CM'$, therefore

$$P : Q :: CN : CM. \quad \text{Q.E.D.}$$

Again, let a force P'', equal to P, act perpendicularly on the lever at M, in a direction opposite to that of P; P and P'' counteract each other and produce no effect on the lever.

Therefore pressure on the fulcrum produced by the four forces P, P', P'', Q, = pressure produced by P' and Q alone — P, P', P'' and Q produce a pressure $P + Q$,

$= P' + Q$, by axiom 2, $= P + Q$,

and pressure produced by P' and $P'' = P' + P''$ by axiom 2, $= 2P$. — P' and P'' produce a pressure $2P$;

Therefore pressure produced by P and $Q = Q - P$. — therefore, P and Q produce a pressure $Q - P$.

Q.E.D.

Cor. 1. Hence, as in the previous proposition,

$$P \times CM = Q \,.\, CN.$$

It is obvious also that the pressure on the fulcrum acts in the direction of the larger force Q.

Cor. 2. If $P : Q :: CN : CM$, where P and Q act on the same side of the fulcrum, P and Q will balance each other.

For let Q' be such a force that it will balance P if (Q') acts perpendicularly at N.

Then $P : Q' :: CN : CM$.

Therefore $Q' = Q$.

But Q' balances P; therefore Q will balance P.

In the same manner we may prove the converse of the other propositions in this chapter.

Prop. IV.

To explain the different kinds of levers.

Strictly speaking there are only two kinds of levers.

(1) Levers, in which the weight and power are on different sides of the fulcrum.

(2) Levers, in which the weight and power are on the same side of the fulcrum.

In both kinds of levers force is gained if the arm at which the power acts is greater than the arm at which the weight acts, and force is lost if it be less.

A poker is an example of a lever of the first kind, the bar of the fender being the fulcrum, the force exerted by the hand being the power, and the resistance of the coals raised, the weight. If the hand be further from the bar than the coals, force is gained; if it be nearer, force is lost.

Scissors, pincers, and snuffers, are double levers of the first kind.

An oar is an instance of a lever of the second kind, where force is gained: the point, where the oar touches the water is the fulcrum, the force exerted by the rower is the power, and the resistance of the water, which is overcome at the rowlock, is the weight.

A pair of nutcrackers is a double lever of the second kind, where force is gained.

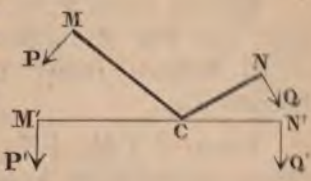
A pair of tongs is a double lever of the second kind, where force is lost. The force exerted by the hand is the power, the resistance which the coals make to the pressure is the weight, and the rivet is the fulcrum.

A knife grinder's wheel, and many of the muscles of the human frame are instances of single levers of the second kind by which force is lost.

Prop. V*.

If two forces acting perpendicularly at the extremities of the arms of any lever, balance each other, they are inversely as the arms.

Let the forces P, Q, acting perpendicularly at M, N, the extremities of the arms CM, CN balance round the fulcrum C.



Then $P : Q :: CN : CM$.

Through C draw $M'CN'$ horizontal,

make $CM' = CM$, $CN' = CN$,

at M' and N' let two forces P', Q' act perpendicularly to the lever such that $P' = P$, and $Q' = Q$.

Construction.
Draw MCN horizontal,
Take
$CM' = CM$
$CN' = CN$
$P' = P$
$Q' = Q$.

Since P, P' are equal forces acting perpendicularly at the extremities of equal arms CM, CM'; by axiom 1, they exert equal efforts to turn the lever round.

And they tend to turn the lever round in the same direction, but P balances Q' by hypothesis, therefore P' will balance Q.

Similarly, since $Q' = Q$ and $CN' = CN$, by axiom 1, Q' and Q exert equal efforts, but Q is balanced by P'.

Therefore Q' is balanced by P'.

And since P' and Q acting perpendicularly on the straight lever $M'CN'$ balance, by Prop. 11.

$$P' : Q' :: CN' : CM'.$$

Q' and P' balance; therefore,
$P' : Q' :: CN' : CM'$.
Hence,
$P : Q :: CN : CM$.

Therefore also $P : Q :: CN : CM$.

* N. B. In the following proposition the arms of the lever are supposed to be straight, but the lever itself is supposed to be bent, that is, the two arms are not in the same straight line.

Prop. VI.

If two forces, acting at any angles on the arms of any lever, balance each other, they are inversely as the perpendiculars drawn from the fulcrum to the lines of action of the forces.

Let the forces P, Q, acting at any angles on the arms CA, CB of the lever ACB, balance round the fulcrum C.

Then, if CM, CN be drawn perpendicular to AP, BQ, the lines of action of P and Q,

$$P : Q :: CN : CM.$$

Since a force produces the same effect at whatever point in its line of action it be applied, (Chap. I. Sect. 2—8) we may suppose the force P to be applied at M, by means of a rod without weight (CAM), fastened to CA.

Similarly, we may suppose Q to be applied at N, by means of a rod without weight (CNB), fastened to CB.

Then P and Q acting perpendicularly on the straight arms CM, CN, balance; therefore, by Prop. 5,

$$P : Q :: CN : CM. \quad \text{Q.E.D.}$$

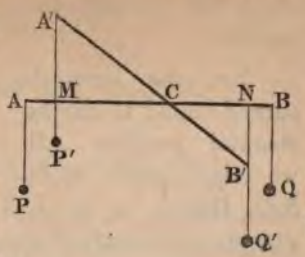
It is clear that in this case the arms AC, CB need not be straight lines.

Cor. Hence $P . CM = Q . CN$.

Prop. VII.

If two weights balance each other on a straight lever when it is horizontal, they will balance each other in every position of the lever.

Let the two weights P, Q balance each other on the straight horizontal lever ACB; they will also balance each other when the lever is turned round into any other position, $A'CB'$.



Construction. Let $A'P'$, $B'Q'$ cut AB in M, N.

Let $A'P'$, $B'Q'$, the vertical lines in which the weights act, (produced, if necessary,) cut AB in M, N.

Then because in triangles $A'MC$, $B'NC$,

Triangles $A'CM$, $B'CN$ are similar.

vertical $\angle A'CM$ = vertical $\angle NCB'$, (I. 15)

and right $\angle A'MC$ = right $\angle CNB'$, (EUC. AX.)

and alternate $\angle CA'M$ = alternate $\angle CB'N$ (I. 27);

therefore the two triangles are equiangular;

therefore $CN : CB' :: CM : CA'$ (EUC. VI. 4); $CN:CM::CB':CA' :: P:Q.$

alternando, $CN : CM :: CB' : CA'$

$:: CB : CA$

$:: P : Q,$

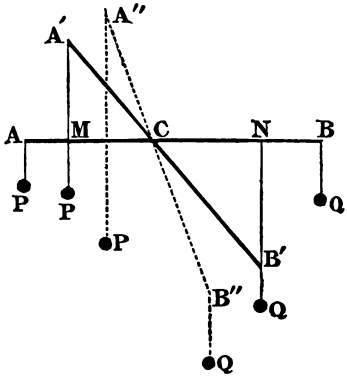
since P balances Q, when AB is horizontal.

Therefore P and Q are inversely as the perpendiculars let fall from C on their lines of action.

Therefore P balances Q when the lever is in the position $A'CB'$. Therefore P, Q, balance in the position $A'CB'$.

COR. Hence we might prove that if P, Q balance in the straight lever in any one position, they will balance in all positions.

For, let them balance in the position $A'CB'$, and let $A''CB''$ be any other position.



Let the lever be turned round to the horizontal position AB; we might prove as before, that

$CB:CA::CN:CM::P:Q$.

Therefore (by Prop. 2) P and Q will balance when the lever is turned into the horizontal position ACB.

Therefore by Prop. 7, they will balance when it is turned into any other position $A''CB''$.

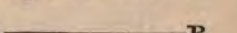
CHAPTER III.

THE COMPOSITION OF FORCES.

If two forces act upon a body at the same instant, and at the same point, we find by experiment, that the body will not move in the direction of either of them, but will move as if it were acted upon by a single force, whose direction is intermediate between the directions of the two forces.

This third imaginary force, which would of itself produce the same effect on the body as the two forces together, is called the resultant of these two forces, and the two forces are called the components of this force.

The force represented by the straight line *AB* in magnitude and direction is called, for the sake of brevity, the force *AB*.



The reader will therefore bear in mind that, when he meets with such expressions as the forces *BC*, *AB*, nothing more is meant than that the forces are represented in direction and magnitude by the straight lines *AB*, *BC*.

Prop. VIII.

If the adjacent sides of a parallelogram represent the component forces in direction and magnitude, the diagonal will represent the resultant force (1) *in direction,* (2) *in magnitude.*

CAF is a right line.

of *AE*, *AB*; therefore *AC* must be in the opposite direction to *AF*; that is, *CAF* must be a right line.

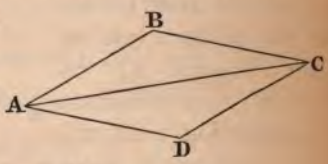
FD is a parallelogram. *AD=FB=AE*. Hence *AD* represents the resultant in magnitude.

Hence *AF* is parallel to *DB*, but *FB* is parallel to *DE* (construction); therefore *FD* is a parallelogram, and *AD* = *FB* (I. 34.) = *AE*, because *EB* is a parallelogram by construction; therefore *AD* represents the resultant of *AC*, *AB* in magnitude.

Prop. IX.

If three forces represented in magnitude and direction by the three sides of a triangle (taken in order) act upon a point, they will keep it at rest.

Let forces represented by the three sides *AB*, *BC*, *CA*, of the triangle *ABC*, act on the point *A*, they will keep it at rest.



Complete parallelogram *BD*.

Complete the parallelogram *BD*.

AD represents in magnitude and direction the force represented by *BC*.

AD = *BC* (I. 34); ∴ *AD* represents in magnitude the force represented in magnitude by *BC*; and *AD* is parallel to *BC*, therefore *AD* represents this force in direction also.

CA, *AB*, *AD* are in equilibrio; ∴ *CA*, *AB*, *BC* are in equilibrio.

Now, *AC* represents the resultant of *AB*, *AD*, but the force represented by *CA* would counteract the force *AC*; therefore it would counteract the forces *AB*, *AD*; therefore it would counteract the forces *AB*, *BC*; therefore the forces *AB*, *BC*, *CA*, would keep the point *A* at rest.

CHAPTER IV.

MECHANICAL POWERS.

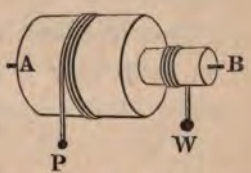
The mechanical powers are contrivances, by means of which a smaller force (called the power) is able to counteract a larger force (called the weight).

The mechanical powers whose properties we are required to investigate are

1. The lever, (for which see Chap. II).
2. The wheel and axle.
3. The pulley.
4. The inclined plane.

DESCRIPTION OF THE WHEEL AND AXLE.

The wheel and axle consists of a cylinder *A*, called the wheel, terminating in a cylinder of smaller base *B*, which is called the axle.



The two cylinders have a common axis, which is supported in a horizontal position.

The power *P* is a heavy body which hangs freely by a string coiled round the wheel.

The weight *W* is another heavy body which hangs freely by a string coiled round the axle.

The power and weight tend to turn the machine round in different directions.

Prop. X.

There is equilibrium on the wheel and axle, when the power is to the weight as the radius of the axle is to the radius of the wheel.

We may suppose P and W to act in the same vertical plane.

The effort of the power to turn the machine round, must be the same at whatever point in the axle the wheel is fixed, we may therefore suppose it to be placed so that the power and weight may act in the same plane, viz. in that plane in which the ends of the wheel and axle both lie. Since the cylinders have a common axis, their circular ends will have a common centre; let O be the centre, join O with M, N the points where the strings leave, and therefore touch, the circle, then OM, ON will be perpendicular to MP, NW. (III. 17.)

O is the common centre of the circles. The angles at M and N are right angles.

Since the axis is at rest, O will be a fixed point, and may be considered as a fulcrum, and we may therefore consider the machine to be a lever moveable round the fulcrum O.

The machine may be considered as a lever. And there is equilibrium if $P:W::ON:OM$.

Therefore there will be equilibrium if

$$P : W :: ON : OM. \quad \text{(Prop. v.)}$$

:: radius of axle : radius of wheel.

Cor. 1. Since MP, NQ are both vertical, MO, ON will be in one and the same horizontal line. Therefore when P and Q act on opposite sides of the axis, the pressure on the fulcrum is equal to $P + Q$ by Prop. II.; if they act on the same side of the axis the pressure on the fulcrum is equal to $Q - P$ by Prop. III.

Cor. 2. If there is equilibrium on the wheel and axle,

$P : W ::$ radius of axle : radius of wheel.

Take W' such that $P : W' ::$ radius of axle : radius heel.

Then W' will balance P, but W balances P, $W' = W$,

ıd $P : W ::$ radius of axle : radius of wheel.

In the same manner the converse of each of the following propositions may be proved.

THE PULLEY.

The pulley consists of a grooved lar wheel (A) moveable round an axis passing through the centre of the l.

The ends of the axis are fixed in a e (C) called the block. The pulley id to be fixed or moveable according as the block ed or moveable.

In the moveable pulley the weight is attached to block, and the power acts by means of a string ng round the wheel A, and fixed at one end.

No force is gained by the use of a fixed pulley, nly effect that it produces is to change the direc- of the motion.

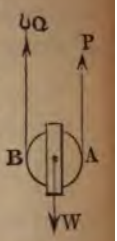
Prop. XI.

In the single moveable pulley, where the strings parallel, there is an equilibrium when the power the weight as 1 *to* 2.

Let the annexed figure represent a single move- pulley where the strings AP, BQ, are parallel.

Each string exerts a force P, $\therefore W = 2P$.

The tension of the string $PABQ$ is the same throughout, or else motion would ensue; therefore each of the strings AP, BQ, must exert the same force; but the string AP is pulled by a force P, therefore the string BQ also exerts a force equal to P.



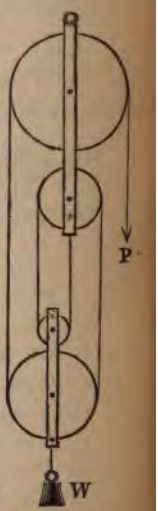
The forces exerted by the two parts of the string support the weight, and they act in the same direction as the weight, therefore they exactly sustain it, and $W = 2P$,

or $P : W :: 1 : 2$.

Prop. XII.

In a system of pullies in which the same string passes round any number of pullies, and the parts of it between the pullies are parallel, there is equilibrium when P : W :: 1 : *the number of strings at the lower block.*

Let the annexed figure represent such a system of pullies.



The tension of the string is the same throughout, or else motion would ensue.

Each string exerts a force P.

Therefore each of the strings at the lower block exerts the same force, but the force exerted by the string to which the power is attached is P, therefore the force exerted by each string is P.

And the sum of these forces supports the weight, but the forces all act in the same direction as the weight, and the sum of the forces must, therefore, be equal to the weight.

$\therefore W = P \times$ number of

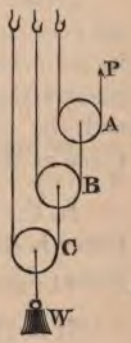
Therefore $W = P \times$ the number of

strings at the lower block, or $P : W :: 1 :$ the number of strings at the lower block. strings at the lower block.

Prop. XIII.

In a system of pulleys where each pulley hangs by a separate string and the strings are parallel, there is equilibrium when P : W :: 1 : *that power of* 2 *whose index is the number of moveable pulleys.*

Let the annexed figure represent such a system of pulleys.



Since A is a single moveable pulley, the weight supported by $A = 2P$;

$\therefore$ tension of string $AB = 2P$. Tension of $BA = 2P$.

Since B is a single moveable pulley, the weight supported by $B = 2 \times$ tension of $AB = 2^2 P$;

$\therefore$ tension of string $BC = 2^2 P$, Tension of $BC = 2^2 P$.

and the weight supported $= W = 2 \times$ tension of $BC = 2^3 P$.

Therefore when there are three pullies $W = 2^3 P$. If there are three pullies, $W = 2^3 P$.

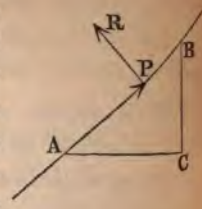
Similarly, we may shew that if there be n pullies $W = 2^n P$. Similarly, if there are n pullies, $W = 2^n P$.

$\therefore P : W :: 1 : 2^n$

$:: 1 :$ that power of 2 whose index is the number of moveable pullies.

THE INCLINED PLANE.

A plane inclined to the horizon is sometimes used as a mechanical power. The plane is supposed to be perfectly rigid and immoveable so as to be capable of counteracting any force which acts upon it in a direction perpendicular to its surface.



Hence if a body is supported on an inclined plane, we may suppose it to be acted on by three forces, viz. its own weight acting vertically, the resistance of the plane acting in a direction perpendicular to the plane, and the power, which must always act in the same plane as the other two forces.

In the case which we are required to consider, the power is supposed to act in a direction parallel to the plane; that is, in the line *AB*.

If in this line any two points *A*, *B* be taken, and we draw *AC* horizontal, and *BC* vertical, so as to meet in *C*,

AB is called the length of the plane,

BC its height,

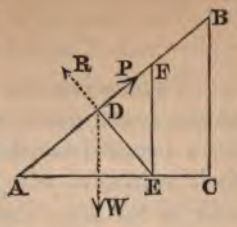
and *AC* its base.

Prop. XIV.

The weight W *being on an inclined plane, and the force* P *acting parallel to the plane, there is equilibrium when* P : W :: *the height of the plane* : *its length.*

Let AB be the length of the inclined plane ABC, BC its height, AC its base.

Let the weight W act vertically at D, and let the power P act on W in the line DB which is parallel to the plane.



There is equilibrium if $P : W :: BC : AB$.

Construction. Draw $\{DE \perp AB, EF \perp AC$.

From D draw DE perpendicular to AB meeting AC in E. From E draw EF perpendicular to AC meeting AB in F.

Then in triangles DEF, ABC,

Triangles DFE, ABC are equiangular.

$$\left\{\begin{array}{l} \angle FDE = \text{right } \angle = \angle ACB, \\ \text{exterior } \angle DFE = \text{interior } \angle ABC, \quad (1.\ 28.) \\ \therefore \text{ remaining } \angle FED = \text{remaining } \angle BAC. \quad (1.\ 32.) \end{array}\right.$$

and the two triangles are equiangular.

$$\therefore BC : AB :: DF : FE, \quad (\text{VI. } 4.)$$

but by hypothesis $P : W :: BC : AB$.

$$\therefore P : W :: DF : FE.$$

Hence DF, FE, represent P and W in magnitude; and P acts in direction DF, and W in direction FE, therefore they represent them in direction also.

DF, FE, represent P and W in magnitude and direction.

Therefore if we complete the parallelogram, DE will represent in direction the resultant of these two forces.

The resultant of P and W acts perpendicularly to the plane.

But DE is perpendicular to the plane, therefore this resultant will be entirely counteracted by the resistance of the plane, and there will, therefore, be equilibrium.

Therefore there is equilibrium.

The velocity of a body is the degree of swiftness with which it moves.

The motion of a body is swifter or slower according as the space passed over in a given time is greater or less; hence if two bodies move with uniform velocities, their velocities will be as the spaces which they pass over in the same time.

The velocity of a body is, in general, measured by the number of feet which it passes over in a second.

Prop. XV.

Assuming that the arcs which subtend equal angles at the centres of two circles are as the radii.

If P *and* W *balance on the wheel and axle, and the whole be put in motion,*

P : W :: W's *velocity* : P's *velocity.*

Let the annexed figure be a representation of the wheel and axle with P and W acting in the same plane.

The horizontal diameter MN is moved into the position $M'N'$.

Suppose that there was originally equilibrium, and that the machine has been moved round until the diameter MN, which was originally horizontal, has come into the position $M'N'$.

MM', NN', are the spaces described by P and W.

Then MM', NN' will be the spaces described by P and W in the same time.

$P : W :: ON : OM :: NN' : MM'$.

Now $P : W :: ON : OM$, since there was originally equilibrium,

$:: NN' : MM'$, (by the assumed Lemma.)

:: space described by W : space described by P in the same time,

$::$ W's vel. : P's vel.

:: W's velocity : P's velocity.

Prop. XVI.

To shew that, if P *and* W *balance each other on the machines described in Propositions* XI, XII, XIII, *and* XIV, *and the whole be put in motion,*

P : W :: W's *velocity in the direction of gravity*

: P's *velocity* (*in the direction in which it acts.*)

Case 1. The single moveable pulley. (Vid. figure in Prop. XI.)

If *W* be raised one inch, each of the strings *QB*, *PA* will be shortened one inch, therefore *P* must be moved through two inches to keep the string tight.

Now, *P* : *W* :: 1 : 2 since there was originally equilibrium,

:: 1 inch : 2 inches,

:: space described by *W* in the direction of gravity : space described by *P* in the same time.

:: velocity of *W* .....
: velocity of *P*.

Case 2. The system of pulleys in which the same string passes round all the pullies. (Vid. figure in Prop. XII.)

If *W* be raised one inch, each of the strings at the lower block will be shortened one inch.

Therefore in order to keep the string tight, *P* must move through as many inches as there are strings at the lower block.

Now, P : W :: 1 : the number of strings at the lower block,

(since there was originally equilibrium)

:: space described by W : space described by P in the same time,

:: velocity of W : velocity of P.

CASE 3. The system of pullies, where each pulley hangs by a separate string. (Vid. figure in Prop. XIII.)

If W be raised 1 inch, and there are three pullies,

If W be raised one inch, each of the strings round the pulley C must be shortened one inch.

Therefore the pulley B must be raised two inches.

Therefore each of the strings round the pulley B must be shortened 2 inches.

Therefore the pulley A must be raised 2^2 inches.

Therefore each of the strings round the pulley A must be shortened 2^2 inches.

P must move through 2^3 inches.

Therefore P must be moved through 2^3 inches in order to keep the string tight.

If there are n pullies, P must move through 2^n inches.

Similarly, if there were n pullies we might shew that P must be moved through 2^n inches to keep the string tight.

$P : W :: 1 : 2^n$

Now, P : W :: 1 : 2^n, since there was originally equilibrium,

:: 1 inch : 2^n inches,

:: space described by W : space passed through by P in the same time.

:: vel. of W : vel. of P.

:: velocity of W : velocity of P.

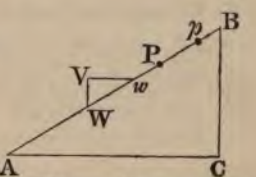
CASE 4. The inclined plane.

In the three former cases W moves vertically. Hence the space which W describes vertically, that is, in the direction of gravity, is the same as the space which it actually passes over, and its velocity in the direction of gravity will be the same as its actual velocity.

This is not the case in the inclined plane, where if W moves at all it must move in the direction of P's action, that is, parallel to the plane.

If P *and* W *balance each other on the inclined plane, and the whole be put in motion,*

$P : W ::$ W's velocity in the direction of gravity : P's velocity.



Let P acting at P, balance W acting at W, on the inclined plane ABC.

Let P pass through the space Pp. W must be moved through an equal space, let this be Ww. Draw WV vertical, wV horizontal, meeting each other in V.

Space described by W in direction of gravity is WV.

The space through which W actually passes is Ww; therefore the space through which it passes in a vertical direction, that is, in the direction of gravity, is WV.

The triangles WVw, ABC, are similar.

In the triangles WVw and ABC

WVw = right $\angle$ = $\angle$ ACB,

alternate $\angle$ Vw W = alternate $\angle$ BAC;

because BA meets the parallels Vw, AC (I. 27.)

$\therefore$ remaining $\angle$ VWw = remaining $\angle$ ABC (I. 32.);

therefore the triangles are equiangular;

BC:AB::VW:Ww. $\therefore BC : AB :: VW : Ww.$ (VI. 4.)

P:W::BC:AB Now, $P : W :: BC : AB$ since there was originally equilibrium.

:: *VW:Ww* or *Pp.* $:: VW : Ww :: VW : Pp.$

:: space described by W in direction of gravity : space described by P.

:: vel. of W in direction of gravity : vel. of P. :: velocity of W in direction of gravity : velocity of P. Q.E.D.

CHAPTER V.

THE CENTRE OF GRAVITY.

DEF. THE centre of gravity of any body or system of bodies is that point, round which the body or system (if acted upon only by the force of gravity) will balance in all positions.

Hence, if the centre of gravity be supported, the body (if acted upon only by the force of gravity) will remain at rest; and if a line which passes through the centre of gravity be supported, (i.e. remain fixed) the body will also remain at rest.

In the definition given above, the particles of the system are supposed to be connected together by inflexible rods, without weight; but the centre of gravity may be conceived to exist when the particles are detached from one another.

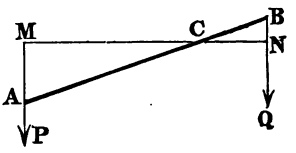
It is evident that in order to possess a centre of gravity a body must possess weight, hence a geometrical point, line, or surface, since it can contain no matter, and can therefore have no weight, cannot have a centre of gravity.

In the following propositions a point is supposed to be, not a geometrical point, but an exceedingly small particle of matter; a line is supposed to be composed of a number of these particles, placed contiguous to each other along the line, and equal in size and weight; and a surface is supposed to be composed of a number of such lines, placed contiguous to each other.

PROP. XVII., forming PROP. XVIII. of the Schedule.

To find the centre of gravity of two heavy points, and to shew that the pressure at the centre of gravity is equal to the sum of the weights in all positions.

Let P, Q be the weights of two heavy points A and B.



Divide AB so that $P:P+Q::BC:AB$.

Let A, B be joined by a rigid inflexible rod without weight.

Divide AB in C, so that $P : P + Q :: BC : AB$,

and therefore dividendo $P : Q :: BC : AC$.

C shall be the centre of gravity of A and B, and the pressure on C shall be equal to $P + Q$.

If AB be turned round C, P and Q balance each other in all positions of the lever; therefore C is the centre of gravity.

If AB were turned round C until it became horizontal, P and Q would act perpendicularly on the lever; and since $P : Q :: BC : AC$, P and Q would balance each other round a fulcrum placed at C (Prop. II.)

And since P and Q balance each other on the lever when it is horizontal, they will balance each other in all positions of the lever (Prop. VII.); therefore (by the definition) C is their centre of gravity.

And pressure on $C = P + Q$.

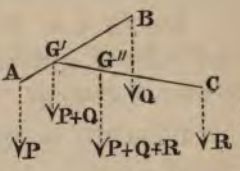
Again, P and Q produce the same effect as if they acted at M and N, and we might shew (as in Prop. VII.) that $P : Q :: CN : CM$; therefore (by Prop. II.) the pressure on C is equal to $P + Q$.

PROP. XVIII., forming PROP. XIX. of the Schedule.

To find the centre of gravity of any number of heavy points, and to shew that the pressure on the centre of gravity is equal to the sum of the weights in all positions.

Let A, B, C be three heavy points, whose weights are P, Q, and R.

Join AB and divide it in G', so that $P : P + Q :: BG' : AB$.



Find G' the centre of gravity of A and B.

Therefore, (by Prop. XVIII.) G' is the centre of gravity of A and B; and, in all positions of the system, they are equivalent to a pressure $P + Q$, acting vertically at G'.

A and B are equivalent to $P + Q$ at G'.

Join $G'C$ and divide it in G'', so that

$$P + Q : P + Q + R :: CG'' :: G'C.$$

Find G'' the centre of gravity of $P + Q$ at G' and C.

Therefore (by Prop. XVIII.) G'' is the centre of gravity of C, and $P + Q$ acting vertically at G'; hence it is also the centre of gravity of C, B, and A.

Also, by the same proposition, the pressure on $G'' = P + Q + R$.

G'' is the centre of gravity of the system, and pressure on $G'' = P + Q + R$.

In a similar manner we might find the centre of gravity of any number of points, and it is obvious that in all such cases the pressure on the centre of gravity will be equal to the sum of the weights.

The same construction may be extended to any number of points.

COR. Since the above construction is always possible, every system of points, (that is, every body) has a centre of gravity.

Then $Ae : eb :: AE : EB$ (by equiangular triangles Aeb, AEB),

$:: AE : EC$ (since $EB = EC$),

$:: Ae : ec$ (by equiangular triangles Aec, AEC).

$eb = ec$; $\therefore bc$ balances on e.

Hence $eb = ec$, and bc will balance on its middle point e.

Similarly, it might be shewn, that any other of the lines parallel to BC, would in all positions balance round the point in which it cuts AE.

The triangle balances on AE; therefore the centre of gravity is in AE;

Hence the whole triangle, being made up of such lines, will balance round AE in all positions.

Therefore its centre of gravity is in AE (Prop. XVII.)

and in CF.

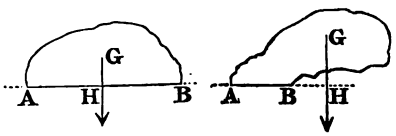
In the same way it may be shewn that the centre of gravity of the triangle is in CF also.

Therefore G is the centre of gravity.

Therefore G, the intersection of CF and NE, is the centre of gravity of the triangle.

Prop. XXII.

If a body rest on a horizontal plane, it will stand or fall, according as the vertical line drawn from its centre of gravity falls within or without the base.



Let the vertical line drawn through G, the centre of gravity of the body, meet the horizontal plane in H.

Through H draw any horizontal line AB, meeting the boundary of the base in A, B.

The weight of the body is equivalent to a single force acting in the line *GH*, and the plane itself exerts a pressure vertically upwards, at all points where the body is in contact with it.

1. Let *H* fall within the base. In this case there will always be some position of *AB* in which *H* will fall between *A* and *B*. If *H* fall within the base.

If *H* be a point in the body, the resistance of the plane at *H* will counteract the weight of the body, and it will remain at rest. Weight of the body is counteracted by the resistance at *H* or supported by the two fulcrums *A* and *B*.

But if not, the resistance of the plane at *A* and *B*, would have the same effect as two fulcrums placed at those points; and the weight acting at *H* would be supported on the lever *AB*, which rests on the two fulcrums at *A* and *B*. Hence the body will still remain at rest.

2. Let *H* fall without the base. In this case there will be no position of *AB* in which *H* will fall within *A* and *B*. But if *H* fall without the base, the weight of the body turns *AB* upwards round *B*, and the body will turn over at *B*.

The weight therefore of the body will not be counteracted as in the former case, but will tend to turn *AB* upwards round *B*, which tendency is in nowise counteracted by the resistance of the plane. Hence the body will turn over at *B*.

Prop. XXIII.

When a body is suspended from a point round which it can swing freely, it will rest with its centre of gravity in the vertical line passing through the point of suspension.

Let the body CG be in any position. Let C be the point of suspension, G the centre of gravity. Draw CN horizontal, and GN vertical, meeting in N.

Draw CN horizontal, GN vertical.

The weight of the body is equivalent to a single force acting vertically at G.

Weight of body acts at G on the arm CN, and will turn the body round C.

Now C is a fixed point, and may be taken as a fulcrum, and the weight of the body will produce the same effect as if it acted N on the arm CN.

There is nothing to counteract this force, a therefore motion will ensue.

But if G be in the vertical line passing through i.e. if CG be vertical, CN vanishes, and the weight the body, acting in the line CG, is entirely cou teracted by the resistance of the fixed point C.

But if CG be vertical, CN vanishes, and weight of body is supported by the resistance of the fixed point C.

Hence the body will remain at rest.

HYDROSTATICS.

CHAPTER I.

1. The science of Hydrostatics investigates the laws which relate to fluid bodies in a state of rest.

2. A fluid is a material body, whose particles can be moved among one another by any force however small, and transmit any pressure impressed upon them, to every other particle with which they may be in contact.

3. Fluids may be divided into elastic and non-elactic fluids.

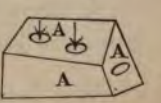
4. Elastic fluids are those whose dimensions are increased or diminished, according as the pressure upon them is increased or diminished.

5. Non-elastic fluids are those whose dimensions are not altered by altering the pressure upon them.

Prop. I.

Fluids press equally in all directions.

Let any close vessel *AA* be filled with fluid, and let any number of equal portions of its surface be removed and their places supplied by pistons exactly fitting the orifices, and acted on by pressures just sufficient to keep them at rest.



It is found by experiment, that if any additional pressure be applied to one of the pistons, an equal additional pressure must be applied to each of the other pistons, in order to prevent them from starting out.

This proves that a pressure applied to any portion of the surface of a fluid at rest is transmitted by the fluid undiminished to every other portion of the surface.

And, we also find by experiment, that whatever be the direction in which any piston is inserted in the vessel, the same pressure is required to keep it at rest.

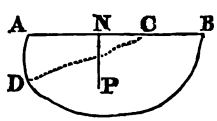
Therefore fluids press equally in all directions.

[We must observe, that in this and the following propositions of the first chapter, the pressure of the atmosphere on the surface of the fluid is neglected, the reasoning being the same as it would be if the experiments took place in vacuo.]

Prop. II.

The pressure on any particle in a fluid is proportional to its depth below the surface.

Let *P* be any particle in a fluid.



1. Let the vertical line *PN*, which meets the surface in *N*, fall entirely within the fluid.

We may suppose the line *PN*, to consist of a number of equal particles arranged along the line.

Now, each particle in PN, receives the pressure of the particle above it, and transmits this pressure, increased by its own weight, to the particle below it.

1. Let P be directly below the surface. Each particle transmits the pressure above + its own weight.

Therefore the pressure on P is equal to the weight of the particles in the line NP.

And since the particles in NP are equal, the pressure on P varies as (or is proportional to) the length of NP.

Therefore pressure at P = weight of particles in NP and $\propto$ NP.

2. Let the line NP not fall entirely within the fluid.

Suppose a part ADC of the fluid to be separated from the rest by a rigid film DC; the equilibrium will not be disturbed, since the rigidity of DC counteracts the upward pressure of the fluid.

Let ADC be enclosed in a rigid film, equilibrium will still remain.

If therefore we remove the portion ADC entirely, and suppose DC to be the side of the vessel, the equilibrium will not be altered, and the pressure on P will be the same as before.

Therefore removing ADC entirely, pressure on P is the same as before, and $\propto$ NP.

Therefore the pressure on P varies as (or is proportional to) NP.

Therefore, in both cases, the pressure on P is proportional to NP, its depth below the surface.

Prop. III.

The surface of a fluid at rest is horizontal.

Let P, Q be any two contiguous particles in the interior of a fluid at rest, situated in the same horizontal plane.

P, Q, are two contiguous particles in same horizontal plane, draw Pa, Qb vertical.

Let the vertical lines Pa, Qb be drawn, meeting the surface a, b.

The vertical pressure on P, Q, have no tendency to move them in the horizontal plane P, Q.

The vertical pressures on P and Q, which act perpendicularly to the plane in which they are, will have a tendency to move them upwards or downwards *out* of this plane, but will have no tendency to move them *in or along* the plane.

The only forces which have a tendency to move them in the plane are the horizontal pressures of the surrounding fluid; and since the particles P, Q are at rest, these pressures must counterbalance each other.

Therefore, since P, Q are at rest, the horizontal pressures on them are equal.

Therefore horizontal pressure on

P = horizontal pressure on Q.

Therefore the vertical pressures are equal; $\therefore Pa = Qb$.

But vertical pressure on

P = horizontal pressure on P.—Prop. I.

= horizontal pressure on Q.

= vertical pressure on Q.—Prop. I.

And these pressures are as Pa, Qb, therefore $Pa = Qb$.

Therefore, $ab \parallel PQ$ and is horizontal.

And Pa is also parallel to Qb, therefore (I. 34) ab is parallel to PQ, and will therefore be horizontal, since PQ is horizontal.

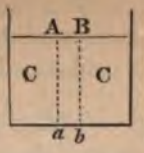
Therefore the surface is horizontal.

Therefore the line joining any two adjacent particles on the surface of a fluid at rest is horizontal; therefore the whole surface is horizontal.

Prop. IV.

If a vessel, of which the bottom is horizontal and the sides vertical, be filled with fluid, the pressure upon the bottom will be equal to the weight of the fluid.

Let *C*, *C* be such a vessel.



We may suppose fluid made up of a number of vertical lines of particles.

We may conceive the whole fluid contained in the vessel to be made up of a number of vertical lines of particles, such as *Aa*, *Bb*.

The pressure produced on the bottom by each of these lines is its weight.

Pressure of each line is equal to its weight.

Therefore the pressure produced on the bottom by all these lines together will be the sum of their weights.

That is, the pressure produced on the bottom of the vessel by the whole fluid will be equal to its weight.

Therefore, whole pressure of fluid is equal to its weight.

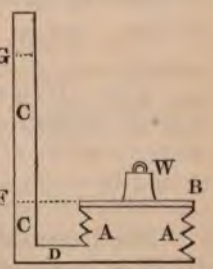
Prop. V.

To state and explain the hydrostatic paradox.

The hydrostatic paradox is this, that any quantity of fluid, however small, may, by means of its weight be made to balance any weight, however large.

A is a vessel whose top *B* is a horizontal board, connected with the bottom by leathern sides, so that it can be raised or depressed.

The vessel communicates with a small vertical tube *C*, by means of a pipe *D*, the tube being of uniform bore.



If water be poured into the instrument it will rise to the same level in the vessel and the tube. Let *BF* be this level.

Water poured into the instrument will rest at the same

level in the vessel and the tube. A small additional quantity of water FG in the tube, will support a large weight W.

If a heavy weight W be now laid on B, a very small additional quantity of water FG will be sufficient to support it.

This is the hydrostatic paradox. We now proceed to explain it.

Whole pressure on area of tube at F equal weight of column FG.

Let A = area of board, a = area of tube, w = weight of column FG; then by 4: pressure on the area a of tube at $F = w$.

Now, since the pressures at each point in the same horizontal plane of a fluid at rest are equal,

The pressures are proportional to the areas.

whole pressure on area a at F : whole pressure on area A of the board :: a : A,

and the pressure on the board = W;

Hence, $w = \frac{a}{A} . W$.

$$\therefore w : W :: a : A, \text{ and } w = \frac{a}{A} W.$$

Therefore, w may be made as small, or W as large as we like, by increasing A or decreasing a.

Therefore, by diminishing a, or increasing A, we may make w as small as we like, without altering W.

Therefore any quantity of fluid, however small, may be made by means of its weight to support any other weight. Q.E.D.

Cor. If, instead of the weight of the column FG, a small pressure equal to the weight of the column FG, were to act at F by means of a piston, the same effect would be produced.

Now, since the weight of the column FG may be made as small as we like, the pressure on the piston, which is equal to it, may be made as small as we like, (viz. by diminishing the diameter of the tube) without altering W.

Hence, any pressure however small may by means of a fluid be made to balance any other pressure however great.

The hydrostatic paradox is sometimes stated in the above form.

If we plunge a solid body into a vessel filled to the brim with any kind of fluid, part of the fluid must flow over the brim of the vessel.

It is also clear that the fluid which flows over the vessel must be equal in bulk to that part (*A*) of the solid which is immersed.

If the vessel into which the solid is plunged be not filled to the brim, that part of the fluid which in the previous case would flow over the brim, will rise in the vessel, and the quantity of fluid that rises in the vessel, owing to its being displaced by the solid, will still be equal in bulk to that portion of the solid which is immersed, and therefore displaces it.

This portion of fluid is called, the fluid displaced by the body.

If a body be suspended from the scale of a balance by a string, the number of pounds which must be placed in the other scale to support it is equal to the weight of the body.

Now if the body be wholly or partially immersed in a fluid, it will require a less weight in the other scale to support it than before, because the fluid itself exerts a pressure upwards on the body.

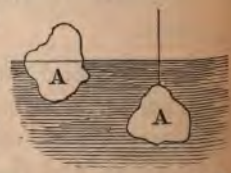
The difference of the two weights that are required to balance the body is called the weight lost by the body.

For instance, if a body weighs 4 lbs. in air, and 3 lbs. when immersed in a fluid, the body would be said to lose 1 lb. in weight.

By this expression it is not meant that the body actually loses 1 lb. in weight, but that it *appears* to do so, because a less weight is required to support it; for if the body be removed from the fluid it will of course weigh 4 lbs. as before.

Prop. (A).

If a body be either wholly or partly immersed in a fluid, the pressure of the fluid upwards on the body is equal to the weight of the fluid displaced.



Let a body A be either wholly or partly immersed in a fluid.

Then the pressure upwards of the fluid on the body is equal to the weight of the fluid displaced.

Supply the place of A with fluid of the same density, and let this fluid become solid, there will still be equilibrium.

If A be removed, and the place which it occupied in the fluid, be filled with fluid of the same density as that in the vessel, there will be equilibrium, and if the portion of fluid thus substituted, become solid without altering its density, there will still be equilibrium.

Now this solidified fluid presses downwards with its weight,

But weight of fluid displaced = weight of solidified fluid.

and weight of fluid displaced = weight of solidified fluid,

= pressure of fluid on the solidified fluid.

..... = pressure of the fluid upwards on the solidified fluid, (since this pressure keeps it at rest),

= pressure of fluid on the body.

..... = pressure of the fluid upwards on the body,

(for the body occupies the same space in the fluid, that the solidified fluid does). Q.E.D.

From this proposition several important corollaries may be deduced.

Cor. 1. *If a body float on a fluid, it displaces as much of the fluid as is equal to the weight of the body; and it presses downwards and is pressed upwards with a force equal to the weight of the fluid displaced.* Prop. VI.

Then the force with which the body presses downwards

= the weight of the body,

= the pressure of the fluid upwards on the body (since this pressure entirely supports the body),

..... = the weight of the fluid displaced (by the previous Prop.)

This corollary forms the sixth Proposition of the Schedule.

N.B. The reader will observe, that in proving this, or the fourth of the following corollaries, he must first prove the proposition itself, on which they depend.

Prop. VII. of the Hydrostatics has already been proved in the first Chapter of Mechanics (viz. Article XII.). Prop. VII.

Cor. 2. *When a body floats in a fluid, the centres of gravity of the body and of the fluid displaced are in the same vertical line.*

The weight of the body is equivalent to a single force acting in the vertical line which passes through its centre of gravity G, and since the pressure upwards of the fluid balances this weight, it must act in the same vertical line.

But this pressure also balances the weight of the fluid displaced, therefore the line of its action must pass through the centre of gravity G' of the fluid displaced.

Therefore G and G' must be in the same vertical line.

Prop. VIII. COR. 3. *When a body of uniform density floats in a fluid, the part immersed : the whole body :: the specific gravity of the body : the specific gravity of the fluid.*

Let a body float with M cubic inches above the fluid, and N cubic inches in it.

Let S be the specific gravity of the body, S' that of the fluid.

Then weight of the fluid displaced $= NS'$
and weight of the body $= (M + N)S$
(by Prop. VII.),

and since the body floats, these two weights are equal, by Prop. VI.;

$$\therefore\ NS' = (M + N)S, \text{ and } \frac{N}{M+N} = \frac{S}{S'},$$

$$\text{or } N : M + N :: S : S'.$$

This forms the eighth Proposition of the Hydrostatics.

Prop. IX. COR. 4. *When a body is* (wholly) *immersed in a fluid, the weight lost : the whole weight of the body :: the specific gravity of fluid : the specific gravity of the body.*

Let M be the magnitude of the body, W its weight, S the specific gravity of the body, S' the specific gravity of the fluid.

Then the weight lost by the body

= the pressure upwards of the fluid upon it,

= the weight of the fluid displaced,

= MS', by Prop. VII.,

and $W = MS$, by the same Prop.;

$$\therefore \frac{\text{the weight lost}}{\text{the weight of the body}} = \frac{MS'}{MS} = \frac{S'}{S},$$

or the weight lost : the weight of the body :: the specific gravity of the fluid : the specific gravity of the body.

This forms Prop. IX. of the Hydrostatics.

Prop. X.

To describe the hydrostatic balance, and to shew how to find the specific gravity of a body by means of it; (1) *when its specific gravity is greater than that of the fluid in which it is weighed;* (2) *when it is less.*

The hydrostatic balance is merely a common balance, with a hook attached to the bottom of one of the scales, so that a body can be weighed not only in air, but also when immersed in some fluid of known specific gravity. The fluid generally employed for this purpose is water. The instrument is used for the purpose of finding the specific gravity of solids.

(1) To find the specific gravity of a body, when it is greater than that of the fluid employed.

Let W be the weight of the body in air, W' its apparent weight when immersed in the fluid.

Let M be the magnitude of the body, S its specific gravity, S' the specific gravity of the fluid.

Then $W - W'$ = the weight lost by the body,

$w - w' \triangleq MS'$. = the weight of the fluid displaced (Prop. A.),

$= MS'$ (Prop. VII.),

$w = MS$. and $W = MS$ (Prop. VII.),

$$\therefore \frac{W}{W - W'} = \frac{MS}{MS'} = \frac{S}{S'},$$

$\frac{w}{w - w'} = \frac{S}{S'}$. or $S = \frac{W}{W - W'} \cdot S'$.

Here S' is supposed to be known, and W, W' may be found by actual weighing, whence S may be determined.

(2) To find the specific gravity of a body P, when it is less than that of the fluid employed.

P will not sink. In this case the body will not sink in the fluid.

Therefore take another body Q as a sinker. Let there be taken therefore any body Q, whose specific gravity is sufficient to sink both itself and P in the fluid.

Let M be the magnitude of P, S its specific gravity, S' that of the fluid; and let w = the weight

f P in air, W = the weight of Q in air, W' = its eight in the fluid, and let w' = the weight of P and) together in the fluid.

Then $W + w - w'$ = weight lost by P and Q together,

= weight of fluid displaced by P and Q,

= weight of a quantity of fluid, whose bulk is equal to that of P and Q together.

$W + w - w'$ = weight of fluid whose bulk is equal to that of $(P + Q)$.

Similarly, $W - W'$ = weight of a quantity of fluid, whose bulk is equal to that of Q.

$W - W'$ = weight of fluid whose bulk is equal to that of Q.

Therefore by subtraction,

$w + W' - w'$ = weight of a quantity of fluid, whose bulk is equal to that of P.

$= MS'$ (by Prop. VII.),

and $w = MS$ (by Prop. VII.);

$w + W' - w'$ = weight of fluid whose bulk is equal to that of P $= MS'$,

and $w = MS$.

$$\therefore \frac{w}{w + W' - w'} = \frac{MS}{MS'} = \frac{S}{S'},$$

therefore, $\frac{w}{w+W'-w'} = \frac{S}{S'}$.

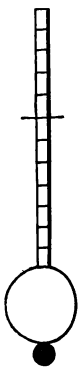
$$\text{or } S = \frac{w}{w + W' - w'} \cdot S'.$$

Here S' is supposed to be known, and w, W', w' may be found by actual weighing, whence S may be determined.

N.B. The body Q is called the sinker.

Prop. XI.

To describe the common hydrometer, and to shew how to compare the specific gravity of two fluids by means of it.



The common hydrometer consists of a hollow sphere, ending in a uniform cylindrical stem; below the sphere is another smaller sphere, which is loaded with shot or mercury, so that the instument floats vertically when immersed in a fluid.

The axis of the stem passes through the centres of both the spheres, and the upper part of it is graduated by divisions of equal lengths. The instrument is always made lighter than the fluids, whose specific gravities it is intended to compare.

To compare the specific gravities of two fluids by means of the hydrometer.

In both cases the weight of the fluid displaced, is equal to the weight of the instrument; therefore they are equal to each other.

Suppose each of the divisions of the stem to be one four thousandth part of the bulk of the whole instrument, and suppose that, when the instrument is placed in a fluid whose specific gravity is S, 10 divisions are above the surface, and that, when it is placed in a fluid whose specific gravity is S', 30 divisions are above the surface.

Then $(4000-10)\,S$ = the weight of the first fluid displaced,

= the weight of the instrument (by Prop. VI.),

= the weight of the second fluid displaced (by Prop. VI.),

$= (4000 - 30)\,S'$,

Hence we get an equation from which S may be determined.

or $3990\,S = 3970\,S'$.

Hence if S' be known, S may be determined.

CHAPTER II.

ON ELASTIC FLUIDS.

THE previous propositions depend upon properties which are common to all fluids, whether elastic or non-elastic. In the present chapter we shall proceed to enunciate and prove some properties which are peculiar to elastic fluids, as well as to discuss and explain certain instruments which depend upon these properties.

We have before stated that fluids are called elastic, when their dimensions are increased or diminished according as the pressure upon them is increased or diminished; that branch of the science which treats of the properties of such fluids is sometimes called by the distinctive name of Pneumatics.

In the succeeding propositions atmospheric air is adopted as the representative of elastic fluids generally, because it is that elastic fluid with which we are most familiar, and whose properties are of the greatest importance to us; but the reader will see that all the conclusions at which we arrive with respect to air, are equally applicable to any other elastic fluid.

Concerning this substance, we might at first feel a doubt whether it be matter or not, inasmuch as under ordinary circumstances it is imperceptible to any of our senses. We may however prove that it is matter, by shewing that it possesses weight.

Prop. XII.

Air has weight.

This may be proved by experiment.

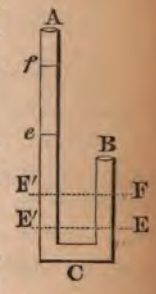
If a close vessel be weighed when full of air, and again when the air has been exhausted from it by means of the air-pump, it will be found to weigh less in the latter case than in the former; and if into a vessel full of common air, more air be forced by means of a condenser, the vessel will weigh more than at first.

These experiments prove that air has weight.

Prop. XIII.

The elastic force of air at a given temperature varies as the density.

A glass tube, of uniform bore, having the end *B* closed, and the end *A* open, is bent into the form *ABC*, so that the two legs are parallel.



A quantity of mercury is poured in through the open end *A*, so as to confine a portion of air in the leg *BC*.

The instrument is then placed with its legs vertical in an exhausted receiver, and the points *E*, *e* at which the mercury rests in the two tubes are marked.

More mercury is then poured in through A, and the points *F*, *f* at which the mercury rests in the two tubes are again marked.

If FF', EE' be drawn horizontal, meeting the other leg in F', E',

we find that $\frac{BF}{BE}$ is always equal to $\frac{eE'}{fF'}$.

Let a be the area of a horizontal section of the tube.

Then weight of mercurial column fF',

= pressure downwards on area a at F', Weight of columns fF', eE'.

= pressure upwards on area a at F' (Prop. I.)

= pressure upwards on area a at F (Prop. II.),

= pressure downwards on area a at F (Prop. I.),

= elastic force of air in BF on area a.

Similarly, weight of mercurial column eE'

= elastic force of air in BE on area a. = elastic force of air in BF, BE.

$$\therefore \frac{\text{elastic force of air in } BE}{\text{elastic force of air in } BF} = \frac{\text{weight of mercurial column } eE'}{\text{weight of mercurial column } fF'},$$

Hence, $\frac{\text{elastic force of air in } BE}{\text{elastic force of air in } BF}$

$= \frac{eE'}{fF'}$ (since both the density of mercury and the bore of the tube are uniform), $= \frac{eE'}{fF'}$,

$= \frac{BF}{BE}$ (by the experiment), $= \frac{BF}{BE}$.

$= \frac{\text{content of } BF}{\text{content of } BE}$ (since the tube is of uniform bore),

$= \frac{\text{density of air in } BE}{\text{density of air in } BF}$ (since the quantity of air $= \frac{\text{density of air in } BE}{\text{density of air in } BF}$,

being the same in both cases, the magnitude varies inversely as the density.) (Mech. Chap. I. 13).

therefore, elastic force ∝ density.

Therefore the elastic force of air varies as the density.

Prop. XIV.

The elastic force of the air is increased by an increase of temperature.

This may be proved by experiment.

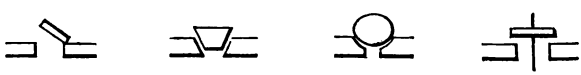
For if a bladder partially filled with air be placed near the fire, the air will expand and distend the bladder; if the bladder be removed to a cooler place the air inside the bladder will cool down, and the bladder will collapse again.

Def. A piston is a plug, which, by means of a rod attached to it, can be made to play within a tube which it fits accurately.

In the construction of instruments, it is very often necessary that a fluid should be able to flow freely through an orifice in one direction, but that it should be hindered from flowing through the orifice in the opposite direction.

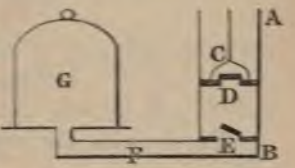
This object is generally effected by the use of a contrivance called a valve, which acts as a kind of door to the orifice, opening when a pressure is applied to it in one direction, and closing when a pressure is applied to it in the other direction.

The annexed are different forms of valves opening upwards.



Prop. XV.

Describe the construction and operation of the common air-pump.



Construction. The air-pump consists of a barrel *AB*, in which a piston *C* works up and down.

In the piston is a valve *D*, and at the bottom of the barrel another valve *E*, both opening outwards; the barrel communicates by means of a pipe *F* (called the suction pipe), with an air tight vessel *G* (called the receiver).

The instrument is used for exhausting the air from the receiver *G*.

Operation. Let us suppose the piston *C* to be at its lowest point, and both valves to be shut.

If piston be at its lowest point and both valves shut.

As *C* ascends, the atmospheric pressure keeps *D* closed, and a partial vacuum is created in the barrel, the pressure therefore is greater on the under side of *E* than on the upper, therefore *E* opens, and the air which previously filled the receiver only, will now expand and fill both the barrel and the receiver.

As the piston ascends, *D* is shut, *E* open, the air in receiver expands, and is therefore rarefied.

Therefore the air in the receiver is rarer than before.

On the descent of the piston, *E* is closed, and the air in the barrel being condensed opens *D* and escapes through it. When the piston again ascends, the air in *G* becomes still more rarefied, and the operation may be continued as long as we please.

As piston descends, *E* is shut, *D* open, and air in barrel escapes through *D*.

NOTE.—Although the piston may be moved up and down the barrel as long as we please, there is a limit to the degree of rarefaction that can be produced; as after a time the elastic force of the remaining air becomes too slight to open the valves; and therefore no more air can be expelled by the ascent of the piston.

PROP. XVI.

To describe the common condenser and its operation.

CONSTRUCTION. The condenser consists of a barrel *AA*, communicating with an air tight vessel *B* (called the receiver), a piston *C* plays within the barrel.

The piston and the bottom of the barrel are furnished with valves, *D*, *E*, both of which open inwards.

The instrument is used for the purpose of compressing a large quantity of air into *B*.

As piston descends, *E* is opened, and *D* closed, and air in *A* is forced into *B*.

OPERATION. Let the piston be at its highest point. As the piston descends, the air in the barrel being compressed will open *E* and keep *D* closed, and the air that was in the barrel will be forced into the receiver *B*.

The air therefore in the receiver will be more condensed than before.

As piston ascends, *E* is closed, and *D* is opened, and external air flows into *A* again.

As the piston ascends, a partial vacuum is created in the barrel, the pressure of the air, therefore, in *B*, will shut the valve *E*, and the pressure of the external air will open *D*, so that it will flow in and fill the barrel again.

At each succeeding descent of the piston more air will be forced into the receiver, and the condensation may be carried on to any extent that may be required.

Prop. XVII.

To explain the construction of the common barometer, and to shew that the mercury in the tube is sustained by the pressure of the air on the surface of the mercury in the basin. (Vid. figure in Prop. xviii.)

1. The barometer consists of a glass tube (*AB*), whose length is about thirty-four inches. One end (*A*) of the tube is open, the other end (*B*) is closed.

The tube is filled with mercury, and then, the end *A* being stopped, it is placed vertically in an open vessel of mercury (*C*), so that the end *A* is a little below the surface of the mercury.

On withdrawing the finger the mercury in the tube subsides, until *A* stands at a height above the mercury in the basin, which varies on different days from 28 to about 32 inches.

The exact height at which the mercury stands, is marked by a graduated scale of inches, at the end *B*.

The instrument is used for measuring the pressure of the atmosphere.

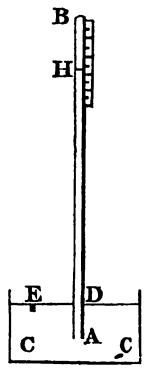
2. The following experiment shews that the mercury in the barometer is sustained by the pressure of the air on the mercury in the basin.

If the instrument be placed under the receiver of an air-pump, as we exhaust the air in the receiver, the

mercury will continue to sink in the tube until it is nearly on a level with the surface of the mercury in the basin. If we readmit the air into the receiver, the mercury will rise in the tube to its former height.

Prop. XVIII.

The pressure of the atmosphere is accurately measured by the weight of the column of mercury in the barometer.



Let the mercury in the barometer rise to the point *H*.

Let *a* be the area of a horizontal section of the tube.

At any point *E* on the surface of the mercury in the basin, take an equal area *a*, and let the tube meet the surface at *D*.

The atmospheric pressure downwards on an area *a* at *E*,

= pressure upwards on an area *a* at *E*, (since there is equilibrium),

= pressure upwards on an area *a* at *D*, (by Prop. II., since *D* and *E* are in the same horizontal plane),

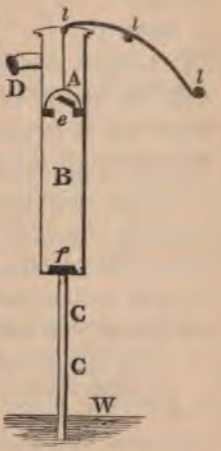
= pressure downwards on an area *a* at *D*, (by Prop. I.),

= weight of mercury in *HD*, (by Prop. IV.)

Therefore the atmospheric pressure on any area is equal to the weight of a column of mercury, whose base is equal to that area, and whose height is equal to the height of the mercury in the tube.

Prop. XIX.

To describe the construction of the common pump and its operation.



Construction. A piston *A*, worked by a lever *lll*, plays up and down in a hollow cylinder *B* (called the body of the pump). *B* communicates with a smaller cylinder *CC* (called the suction pipe), whose length must be less than 34 feet.

In the piston and at the bottom of *B* are valves *e*, *f*, both opening outwards, and a little above the highest point to which the piston ascends is an open spout *D*. The end of the suction pipe is placed a little below the surface of a reservoir of water *W*; and the machine is used for the purpose of raising the water in *W* through the spout *D*.

Operation. Let the piston be at its highest point, and both valves shut.

Let the piston be at its highest point, and both valves shut. As the piston descends, *f* is shut, *e* open, and air in *B* escapes through *f*.

As the piston descends, the air in *B* being compressed, closes *f*, and opens *e*, and the air which was in *B* escapes through *e*. When the piston has descended to its lowest point, *e* shuts by its own weight.

When the piston ascends, a partial vacuum is created in *B*, *e* therefore is kept closed by the pressure of the external air, and the air in *CC* presses against and opens *f*, and expands so as to fill *B*, as well as *C*. When the piston has ascended to its highest point, *f* shuts by its own weight.

As the piston ascends, *e* is shut, *f* open, and the air in *C* expands and fills the barrel.

Thus the air in *c* is rarefied, and its pressure is decreased; therefore atmospheric pressure forces the water up *C*.

The air therefore in *C* becomes less dense, and therefore exerts a smaller pressure on the surface of the water in *W* than it did before; it will therefore no longer balance the pressure of the external air on the surface of the water in *W*, and this pressure will therefore force the water up the tube *CC*, until equilibrium is restored.

At each ascent of the piston, the water ascends in *C* until it flows through *f*, when the next ascent of the piston lifts it through *D*.

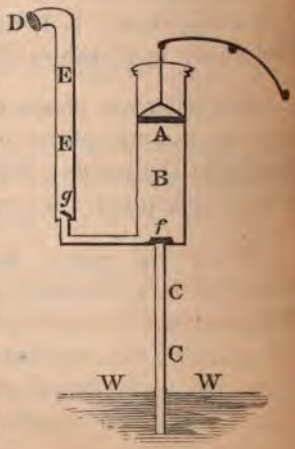
At each successive ascent of the piston the water will rise higher in *CC*, until at last it flows through *f*, when the next ascent of the piston will lift it so as to flow through the spout *D*.

(The height of the valve *f* above the surface of the water in the well, must be less than the height of the water barometer, or else the pressure of the external air will not be able to raise the water up to *f*; and the pump will not work.

The height of the water barometer in this country is generally about 34 feet.)

Prop. XX.

To describe the construction of the common forcing pump and its operation.



Construction. A solid piston *A* works up and down a cylinder *B*, by means of a rod attached to a lever; *B* communicates with a suction pipe *CC*, the end of which is a little below the surface of a reservoir of water *W*; a pipe *EE* ending in a spout *D*, opens into the bottom of the cylinder *B*, and where *CC* and *EE* join the cylinder *B* are valves *f*, *g*, opening outwards.

instrument is used for the purpose of raising
r from *W* through *D*, and the tube *EE* may
y height that is required.

RATION. Let the piston be at its highest
ıd both valves closed.

he piston descends, the air in *B*, being com-
shuts *f*, and opens *g*, and the air in *B* escapes
g.

he piston ascends, a partial vacuum is created
ıe pressure of the external air will therefore
and the air in *CC* will open *f*, and expand so
both the tube *CC* and the barrel *B*.

As piston ascends, *g* is shut, and *f* open, and the air in *CC* expands, and fills the barrel.

air therefore in *CC* will be rarefied, and its
on *W* will be decreased, and will therefore be
o balance the pressure of the atmosphere.

Therefore its pressure is decreased, and the pressure of the atmosphere forces the water up *C*.

atmospheric pressure will therefore force the
) *CC* until equilibrium is restored.

ach ascent of the piston the water will rise in
til at last it flows through *f*, when the next
of the piston will force the water through *g*.

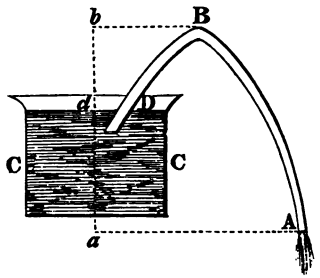
When the water flows through *f*, the descent of the piston forces it through *g*, whence it cannot return, and each descent of the piston forces it up higher in *EE*, until it flows through *D*.

when the piston ascends *g* is closed, therefore
r cannot return through *g*, and each successive
of the piston will force the water a little higher
until at last it flows through *D*.

in the case of the common pump, the height
alve *f*, above the surface of the water in *W*,
less than 34 feet.)

Prop. XXI.

To describe the siphon and its operation.



The siphon consists of a bent tube *ABD*, of uniform bore, and with legs of unequal length.

It is used for emptying fluids from a higher to a lower level.

The shorter leg is placed beneath the surface of the fluid *CC*, the end *A* is stopped, and the air is exhausted from the tube by means of suction, or the air-pump, when the atmospheric pressure forces the fluid up the tube, until it fills the siphon (provided the height *bd* of *B* above the surface of the fluid, is not greater than the height of a column of fluid, whose weight just counterbalances the atmospheric pressure). On opening *A* the fluid will flow through the tube, and out of *A*.

To shew the reason of this.

Let the shorter leg meet the surface in *D*, and let *a* equal the number of square inches in a horizontal section of the tube. When *A* is unstopped, the fluid columns *BD*, *BA* have a tendency to fall by their own weight, leaving a vacuum at *B*, but (provided the height of *B* above the surface is not greater than that of a barometer formed of the same fluid) the atmospheric pressure will sustain the column *BD*, and the continuity will not be interrupted.

If *Bd* be not greater than the height of a barometer formed of the same fluid, the continuity will not be interrupted, and fluid in siphon will act in the same way as a heavy chain.

We may therefore argue concerning the fluid in the siphon, as if it were a heavy chain.

Now the forces which act are :—

The forces acting are atmospheric pressures at A and D, which balance each other; and the fluid columns BD, BA; therefore the fluid will flow in the direction of the longer leg.

(1) The atmospheric pressure on the area a at A, acting from A to D.

(2) The pressure of the fluid upwards on the area a at D, acting from D to A.

And this = the pressure of the fluid upwards on any equal area a, in the surface of the fluid,

= atmospheric pressure on area a on the surface,

and the atmospheric pressure being equal at these points, these two forces must counteract each other.

The only remaining forces are :—

(3) Pressure of fluid on BD, which

= weight of a column of fluid whose base is a, and height bd.

(4) Pressure of fluid on BA, which

= weight of a column of fluid whose base is a, and height ba.

Therefore if $Ba > Bd$, the fluid will flow through A.

Note. The pressure of the atmosphere at D is not the same as its pressure at a, (by Prop. II.) since the point a is lower than D; but, the density of the air being very small, the difference between the atmospheric pressures at these two points is so slight that it may be neglected, and the two pressures may be considered as practically equal.

Prop. XXII.

To shew how to graduate a common thermometer. (Vid. figure in Prop. XXIII.)

The thermometer consists of a glass bulb, communicating with a slender glass tube, of uniform bore.

The end of the tube being open, the bulb is filled with mercury, and placed over a spirit lamp, until the mercury rises in vapour through the tube, when the flame of a blow-pipe is passed over the end, and hermetically seals it by melting the glass.

There will therefore be a vacuum in the tube above the surface of the mercury.

The instrument is then placed in melting snow, and the point to which the mercury sinks (A) is marked as the freezing point.

The instrument is next placed in boiling water, and the point to which the mercury rises (B) is marked as the boiling point.

The intermediate space is divided into any number of equal degrees, and the parts of the thermometer above B and below A, are very often graduated according to the same scale.

The instrument is used for measuring the temperatures of different bodies.

Prop. XXIII.

Having given the number of degrees on Fahrenheit's thermometer, to find the corresponding number on the centigrade thermometer.

In Fahrenheit's thermometer the freezing point is marked 32^0 and the boiling point 212^0, so that the space AB, between the two contains 180^0 graduations.

In the centigrade thermometer the freezing point is marked 0, and the boiling point 100^0, so that the space AB contains 100^0 graduations.

Let the mercury stand at any point Q.

Then, since the graduation is uniform in both cases,

Number of degrees Fahrenheit in AQ : number of degrees Fahrenheit in AB :: number of degrees centigrade in AQ : number of degrees centigrade in AB.

Let C^0 be the number of degrees marked opposite to Q on the centigrade scale, F^0 the number of degrees marked opposite to Q on the Fahrenheit scale.

$\therefore$ $F^0 - 32$ = the number of degrees Fahrenheit in AQ,

and $F^0 - 32 : 180 :: C^0 : 100$,

or $100\ (F^0 - 32) = 180\ C^0$.

From which equation we may find C^0, when F^0 is given; or F^0, when C^0 is given.

NOTES.

NOTES.

DEF. 1. THUS it would belong to the province of Statics to shew by what forces a boat may be kept at rest in a current, or a ball be supported on an inclined plane; if the boat were put into motion, or the ball allowed to roll, it would belong to the province of Dynamics to explain all the circumstances relating to the motion.

DEF. 2. Whatever causes, or, if not counteracted, would cause, the state of a body's rest or motion to be different from what it would otherwise be, is called a force.

There are some forces, whose only effect is to retard motion; that is, to keep a body in a state of rest, or to reduce a moving body to a state of rest; such forces are called forces of resistance. The friction of surfaces, the resistance of the air, or water, or that of an immoveable point, such as the fulcrum of a lever, are forces of this kind.

We know from experience, that, when a heavy body is at rest on a rough surface (as is the case when a coach stands on a road), it requires a considerable effort to move the body. If we push against it slightly it will not stir, although we are conscious that, in pushing against it, we exert a force. In all such cases we call into action a new force, viz., the force of friction, and this is sufficient to counteract the force which we apply to the body. We find, however, that if we increase the force, which we apply to the body, to a sufficient extent, we are able to overcome the friction, and move the body.

That this resistance to motion is produced by an extraneous force, and is not the result of any natural tendency of the body itself to a state of rest, is proved by the

fact, that if we render the surface on which the body rests smoother than before, a smaller force will be sufficient to move the body. Thus a body which rests on a smooth platform can be moved more easily than if it rested on a rough road, and if it rests on a sheet of ice, it can be moved with still greater ease; and hence we may conclude that if the surface on which it rested were perfectly smooth, we might move it by any force however small, provided the friction between the surface of the body and the plane on which it rests, were the only resisting force that acts upon the body.

In fact, bodies have in themselves no tendency either to rest or motion. That they have no tendency to motion, is a matter of daily experience, for we all know that a body at rest will not move unless acted upon by some external force; but it is not equally obvious that a body in motion will continue to move for ever with the same velocity, unless it is acted upon by some external force, since all bodies which we see in motion are, if left to themselves, found to move slower and slower, and finally to stop.

In all such cases, however, we find, if we examine the matter carefully, that the body is acted upon by some cause which tends to diminish its velocity; and if we either remove, or make allowance for all external causes, which must affect the motion of the body, such as friction, gravity, the resistance of the air, &c., we find that *a body in motion will continue to move in the same direction, and with a uniform velocity, unless it be acted on by some external force.*

If, therefore, either the direction or the velocity of a body's motion are altered, or if a body is put into motion from being in a state of rest, or if it remain at rest when influenced by causes which would under other circumstances be capable of imparting motion to it, this change must have been produced by some external cause. Whatever causes such a change is called a force.

Thus the air which resists the motion of a projectile,

and the water, which retards the speed of a vessel, are said to exert a force; the resistance which hinders a book from sliding down a plane slightly inclined to the horizon is called a force; and the wall which changes the direction of a ball thrown against it is said to exert a force, as well as the hand by which the ball is thrown, and the earth which attracts it to its surface, and from which by the force of elasticity it again rebounds.

Def. 3, 4, 5. Every particle of matter, however small, attracts every other particle. It may however be proved theoretically, that the united influence of all the particles which compose the earth, is the same as it would be if all those particles were collected at the earth's centre. This theoretical conclusion is confirmed by constant and general experience; for all known bodies, when left to themselves fall perpendicularly to the earth's surface, that is, in the direction of its centre.

The cause which produces this effect must, according to our definition, be called a force, and this force is said to be exerted by the earth; because it causes bodies to fall to the earth.

As the earth attracts all bodies towards itself, so it is attracted towards them in return; but their masses are so small when compared with that of the earth, that the earth's attraction only is apparent to us. For the same reason the mutual attraction of bodies on the surface of the earth, is not in general perceptible to us, although in some cases it has been observed that a plumb-line, which under ordinary circumstances hangs vertically, has been deflected from the vertical by the attraction of a mountain.

Again, if we prevent a body from falling to the earth, by supporting it with our hand, we find that the body exerts a pressure upon it. This pressure exerted by the body is called its weight, and always acts in the direction of the earth's centre. Strictly speaking, the body does not of itself exert a pressure upon the hand; it is the

attraction of the earth which causes the body to press downwards; but as the pressure upon the hand is exerted through the medium of the body, the above expression has been adopted to avoid circumlocution.

It may be objected to this definition that there are certain forms of matter, such as light, colour, &c. which are imponderable. The consideration however, of these substances does not come under the range of what is strictly called Mechanics. Nor can it be proved that these forms of matter, if indeed they be forms of matter, are absolutely without weight; all that can be considered as certain is, that their gravity is so slight that no instruments which we possess are sufficiently fine to detect it. Of all those bodies whose existence is obvious to our senses it may be truly said: "Nulla dari corpora plane levia, jamdudum confirmavit experientia multiplex."

By impenetrability is meant that property of matter, by virtue of which, no two particles can occupy the same point of space at the same time.

A striking example of the impenetrability of matter will be found in page 49.

When we wish to express the magnitude or size of any thing, it is necessary, in the first place, to fix upon some definite and well-known portion of that kind of thing. The portion thus fixed upon is called a unit. We then state the number of times that this portion is contained in the thing in question. If, therefore, we wish to state the magnitude of a weight, our first step must be to fix upon some standard unit of weight, and then we can state how often this standard unit is contained in the weight in question.

We can estimate weight only by means of its mechanical effects, and therefore we naturally conclude that if any two bodies produce equal mechanical effects they possess equal weights. The mechanical effect, which furnishes the most convenient standard for this purpose, is the

bending of a spring to a certain point, inasmuch as this effect is subject to very little variation.

If two bodies bend the same spring down to the same point, we conclude that their weights are equal; if another body bends the spring to the same point that the two first bodies together do, it will weigh twice as much as either of them, and so on. If, therefore, we take a particular portion of some known substance, as for instance $2\frac{1}{2}$ cubic inches of lead, the weight of this portion of lead will serve as the standard unit of weight, and will have some peculiar name, as for instance, a pound, applied to it. Whatever body, by means of its weight, bends the spring down to the same point as this piece of lead, will weigh a pound also; and whatever body bends the spring down to the same point as 2.3.4, &c., such pieces of lead, when applied at the same time, will contain 2.3.4, &c., of the standard units of weight, and will weigh 2.3.4, &c., pounds.

Def. 9. The definition of density given here, differs from that which is usually given, because it was necessary to give analogous definitions of density and specific gravity, and no definition of specific gravity but that given in Art. 11, is consistent with the formula $W = MS$, which is one of the propositions required to be proved in the schedule.

Since the densities of different bodies are defined to be the quantities of matter contained in a unit of volume of those bodies, it follows that in bodies of uniform density, the densities will be proportional to the quantities of matter in any equal volumes of those bodies, and therefore, by Def. 7, the densities will be proportional to the weights of any equal volumes of those bodies.

Thus, two bodies possess the same density, when equal volumes of those bodies possess the same weight; if a volume of the one weighs twice as much as an equal volume of the other, the density of the first is said to be twice as great as that of the other; if a volume of the first

weighs three times as much as an equal volume of the other, the first is said to be three times as dense as the other, and so on. It will be seen therefore, from what has been stated above, that density, and quantity of matter, refer merely to the number of particles contained in a body, whilst specific gravity and weight refer to the force exerted by those particles.

Also, since the quantities of matter in different bodies are proportional to their weights, the densities of different bodies are proportional to their specific gravities.

In the tables of specific gravities, the specific gravity of distilled water at a temperature of 60° Fahr., is supposed to be 1, and from this the specific gravities of other bodies are calculated.

It is evident that a table of specific gravities will also serve as a table of densities, if we suppose the density of water at 60° to be 1 as well as its specific gravity.

For the sake of convenience, another standard unit of specific gravity has been adopted in the case of elastic fluids, where 1 is supposed to be the specific gravity of dry atmospheric air, when the thermometer stands at 60°, and the barometer at 30 inches.

DEF. 12. The definition of specific gravity is so intimately connected with that of density, and the formula $W = MS$ follows so directly from the definition, that we have thought it advisable to place them immediately after the definition of density, rather than in the place in which they stand in the schedule.

DEF. 13. This article contains the proof of a principle which is used in Prop. 13, of the Hydrostatics, and will scarcely appear self-evident to a beginner.

Section II. 1. There is a difference between the statical and dynamical measures of force, because in Dynamics we have to consider not only the mass which is moved, but also the velocity with which it is moved.

The measure of a dynamical force is its momentum, that is, the product of the mass (which is moved) and the velocity (with which it is moved).

We must distinguish between the magnitudes of forces, and the ratios which those magnitudes bear to each other. It is only the ratios that can be represented by right lines.

For instance, if the forces P, Q, R, were represented by three lines of which the first was one half of the second, and the second one half of the third, we should know that Q was equal to $2P$, and R equal to $2Q$, but we should still be ignorant whether the forces were 1, 2, and 4 pounds, or 1, 2, and 4 cwt. If, however, the magnitude of one of the forces were known, the magnitude of the others might be determined from the ratios; or if we agreed to represent a certain amount of force, as, for instance, a pound, by a line of a certain length, as an inch, then any line whose length was two inches would represent two pounds, and so on.

EXAMPLES.

1. A PINT of water weighs 1lb., a pint of gold 19lbs.; compare the densities and specific gravities of water and gold.

Since by Article X. the densities of different bodies are proportional to the weights of any equal volumes of those bodies,

Density of water : density of gold :: 1 : 19,

and the densities of different bodies are proportional to their specific gravities;

$\therefore$ specific gravity of water : specific gravity of gold :: 1 : 19.

The same thing might be proved from the formulæ in Art. 12, for if M be the volume of a pint, that is, the number of cubic inches contained in it; D, D' the densities

of water and gold, S, S' their specific gravities, Q, Q' their masses, W, W' their weights,

$$\frac{S}{S'} = \frac{MS}{MS'} = \frac{W}{W'} = \frac{1}{19},$$

$$\frac{D}{D'} = \frac{MD}{MD'} = \frac{Q}{Q'} = \frac{W}{W'} \text{ (by Art. 7)} = \frac{1}{19}.$$

2. What relations exist in bodies of the same magnitude, between their weights, masses and densities?

In this case we have for any two bodies,

$$Q = MD,\ Q' = MD', \text{ and } \frac{Q}{Q'} = \frac{W}{W'}, \text{ (Art. 7).}$$

$$\therefore \frac{Q}{Q'} = \frac{MD}{MD'} = \frac{D}{D'},$$

or the mass varies as the density;

$$\text{and } \frac{W}{W'} = \frac{Q}{Q'} = \frac{D}{D'},$$

or the weight varies as the density.

3. If a cubic foot of cork weighs 15 lbs. and a cubic foot of tin weighs 450 lbs., what will be the density of tin, the density of cork being .24?

$$\frac{\text{density of tin}}{\text{density of cork}} = \frac{450}{15} = \frac{30}{1},$$

$\therefore$ density of tin = 30 × density of cork = 30 × .24

= 7.2.

4. Five pints of alcohol weigh 4 lbs., and 10 quarts of oil weigh 18 lbs., compare the specific gravities of oil and alcohol,

as 5 pints : 10 quarts :: 4 lbs. : the weight of 10 quarts of alcohol.

$\therefore$ the weight of 10 quarts of alcohol $= \frac{20 \times 4}{5}$ lbs. $= 16$ lbs.

$\therefore$ $\frac{\text{specific gravity of oil}}{\text{specific gravity of alcohol}} = \frac{\text{weight of 10 quarts of oil}}{\text{weight of 10 quarts of alcohol}}$

$$= \frac{18}{16} = \frac{9}{8}.$$

5. If 8 cubic feet of fir weigh 275 lbs., and 72 cubic inches of hazelwood weigh 25 oz., find the specific gravity of fir, that of hazelwood being .6,

as 72 cubic inches : 8 cubic feet :: 25 oz. : weight of 8 feet of hazelwood;

$\therefore$ weight of 8 feet of hazelwood $= \frac{8 \times 1728 \times 25}{72}$ oz.

$= 4800$ oz. $= 300$ lbs.,

and $\frac{\text{specific gravity of fir}}{\text{specific gravity of hazelwood}} = \frac{275}{300} = \frac{11}{12}$.

$\therefore$ specific gravity of fir $= \frac{11}{12} \times$ specific gravity of hazelwood

$$= \frac{11}{12} \times .6 = \frac{6.6}{12} = .55.$$

6. Compare the quantities of matter in a piece of wood which weighs 3 cwt., and a block of marble which weighs 2 tons.

Since the quantity of matter in different bodies is proportional to their weights,

$$\frac{\text{the quantity of matter in the wood}}{\text{the quantity of matter in the marble}} = \frac{3 \text{ cwt.}}{2 \text{ tons}} = \frac{3}{40}.$$

7. If a cubic foot of water weigh 1000 oz., and the density of manganese be to that of water as 8 : 1, what will 54 cubic inches of manganese weigh?

(By Art. 7), $\frac{\text{weight of a cubic foot of manganese}}{\text{weight of a cubic foot of water}} = \frac{8}{1}$.

$\therefore$ weight of a cubic foot of manganese = 8000 oz.

as 1728 inches : 54 inches :: 8000 oz. : weight of 54 inches of manganese;

$$\therefore \text{ weight of 54 inches of manganese} = \frac{8000 \times 54}{1728} \text{ oz.}$$

$$= \frac{8000}{32} \text{ oz.} = 250 \text{ oz.}$$

8. Compare the quantities of matter in a piece of gold which weighs 15 oz., and a pint of water which weighs 1 lb.

9. When is one body said to weigh 4 times as much as another body?

10. When is one body said to be 4 times as dense as another body?

11. A cubic foot of copper weighs 8900 oz., and a cubic foot of nickel weighs 7666 oz., compare the densities of copper and nickel.

12. A cubic foot of silver weighs 10470 oz., and a cubic foot of manganese weighs 500 lbs. If 10.47 represent the specific gravity of silver, what will be the specific gravity of manganese?

13. Two pounds of brass weigh as much as $1\frac{1}{2}$ lb. of silver, compare the specific gravities of silver and brass.

14. In the preceding example find the specific gravity of brass, the specific gravity of silver being 10.5.

15. If 6 cubic feet of manganese weigh 3000 lbs., and 216 cubic inches of zinc weigh 875 oz., find the specific gravity of zinc, the specific gravity of manganese being 8.

16. The specific gravity of platinum is 21.5, that of silver 10.5, how many cubic inches of platinum weigh as much as $14\frac{1}{3}$ cubic inches of silver?

17. The specific gravities of zinc and manganese are 7 and 8. How many cubic feet of zinc weigh as much as $10\frac{1}{2}$ cubic feet of manganese?

18. If 2 cubic feet of black lead weigh 300lbs., and the density of black lead is to that of common lead as 40 : 189, what will $36\frac{4}{7}$ inches of common lead weigh?

19. If 5 pints of oil weigh $4\frac{1}{2}$ lbs., what will be the weight of 3 quarts of alcohol, the specific gravity of oil being to that of alcohol as 9 : 8.

20. Four cubic inches of gold whose specific gravity is 18, is mixed with 6 cubic inches of silver whose specific gravity is 10. Find the specific gravity of the compound.

Let x = the specific gravity of the compound,

$(4 + 6)\,x$ = the weight of the compound,

= the sum of the weights of the gold and silver,

$10x = 4.18 + 6.10 = 132,$

$x = 13.2.$

In the preceding example we have supposed the compound to occupy the same space that the gold and silver together occupied before they were combined. Substances, however, when combined do not necessarily occupy the same spaces that they do when separate.

Thus if a glass be filled to the brim with water, a certain amount of sugar may be dissolved in the water, without its running over the sides of the vessel; and if we mix together a pint of water and a pint of sulphuric acid, the mixture will not make a quart.

21. A pound of lead is mixed with a pound of silver, and the bulk of the mixture is equal to the bulk of the two substances when separate, if the densities of lead and silver are 11.5 and 10.5; what will be the density of the compound?

22. Three pints of water are mixed with 6 pints of alcohol; if there are 9 pints of the mixture, and the specific gravities of water and alcohol are 1 and .8, find the specific gravity of the compound.

23. Four quarts of sulphuric acid are mixed with 4 pints of water; if there are only 10 pints of the mixture, and the specific gravities of sulphuric acid and water are 1.8, and 1, what will be the specific gravity of the mixture?

CHAP. II. Axiom 1. This axiom is manifest from the consideration that, the forces being equal, and acting in the same manner at the extremities of equal arms, no reason can be assigned why the effort exerted by the one should be greater than the effort exerted by the other.

The same reasoning is applicable to Axiom 3, for the fulcrums are similarly situated with respect to the weight supported, and therefore no reason can be assigned why the pressure upon the one should be greater than the pressure upon the other.

The following reason has been assigned for Axiom 2.

The two forces must be supported, and it is evident that the only supporting force is that exerted by the fulcrum; the pressure of the two forces upon the fulcrum will not depend at all upon the length of the lever, but will be the same as if the two arms were shortened till the weights coincided at the fulcrum; but in this case it is clear that the pressure on the fulcrum will be equal to the sum of the weights, therefore it will be so in every other case.

If this axiom be admitted, there seems to be no sufficient reason why we should not at once generalize the principle, and adopt the following as an axiom.

If any number of parallel forces acting in the same direction on the arms of any lever balance round any fulcrum, the pressure of the fulcrum will be equal to the sum of the forces and will act in the same direction as the forces.

Hence it follows, that, if the fulcrum be situated at any point F, a force equal to the sum of the forces, and acting at F in the opposite direction, would serve instead of a fulcrum, and counterbalance them if the fulcrum were removed.

But such a force would also counterbalance a single force, equal to the sum of the forces, and acting at F in the same direction; therefore the parallel forces produce the same effect as, and are therefore equivalent to, a single force equal to their sum, and acting at F in the same direction.

Hence we might readily prove that,

A horizontal cylinder of uniform density will produce the same effect as if collected at its centre.

For if we suppose a fulcrum to be placed at the centre, since the parts of the cylinder on each side of the fulcrum are equal, similar, and similarly situated with respect to the fulcrum, there is no reason why the cylinder should turn to one side rather than to the other, and therefore the cylinder will balance round the fulcrum.

Now the cylinder may be supposed to consist of a number of particles of equal weight equally diffused through the cylinder; and the weights of all the particles produce the same effect as a single weight equal to their sum acting at the point where the fulcrum was placed, viz. at the centre of the cylinder.

If therefore we remove the fulcrum entirely, the whole cylinder will produce the same effect as if all the particles composing it were collected at its centre.

It will be seen, that the reason assigned for axiom 3, if valid in that case, is equally valid with respect to axiom 4.

Nevertheless these two axioms have been considered so unsatisfactory by many that attempts have been made to supersede them.

The following proof of the Lemma of Prop. 1, is to be found in the *Mechanique Analytique* of Lagrange, and is independent of either of these axioms.

Let ABC be any equilateral triangle situated in a horizontal plane, and let a weight $2P$ act at A and weights P, P act at B, C;

bisect AB in d and draw de parallel to BC;

bisect BC in f, and join Af cutting de in g,

then 6.4, $Ad : dB :: Ag : gf :: Ae : eC$;

but $Ad = dA$; $\therefore Ag = gf$ and $Ac = eC$.

P at A and P at B would balance round d,
and, P at A, and P at C would balance round e;

$\therefore 2P$ at A, P at B, and P at C balance round de.

But P at B, and P at C, would balance round f;

$\therefore 2P$ at A, P at B, and P at C balance round Af;

$\therefore$ the three weights will balance round g the point of intersection of Af and de;

but $2P$ at A, would balance $2P$ at f, round the fulcrum g.

Therefore P at B, and P at C, may be counteracted by the same force as $2P$ at f, and they must therefore produce the same effect as $2P$ at f, the middle point between B and C.

A proof of the property of the lever upon similar principles may be found in the works of Huyghens.

PROP. 1. In most treatises upon Mechanics the principles of Statics are deduced from the parallelogram of forces, (see Prop. 8), instead of the property of the lever; but the arrangement of the propositions in the Schedule renders it imperative upon us to prove the property of the lever independently.

It may safely be asserted that students in general find far more difficulty in the two first propositions of the Schedule than any of the rest; nor is this to be wondered at, when in most of the elementary treatises on Mechanics these propositions are made to depend upon definitions which are inadequate, and axioms which are not selfevident.

For instance, in the enunciation of the first proposition we meet with a very singular expression, viz., the middle point of a prism or cylinder. It may be sufficiently obvious that by the middle point of a cylinder, is meant the middle point of its axis; but what, it may be asked, is meant by the middle point of a prism? The expression might be supposed to mean the middle point of the circumscribed cylinder, but unfortunately a cylinder cannot always be described about a given prism, nor, even if it could, would the proposition be necessarily true; neither, for the same reason, can it mean the middle point of the inscribed cylinder; to a beginner therefore the expression is perfectly unintelligible. But it is not necessary that the theorem should be true with respect to a prism, it is sufficient for our purpose, if we can prove it to be true for the cylinder alone.

Now, if we examine the proposition, we find that the proof given in the text depends upon the assumption that, "if we suppose the cylinder to be divided into a number of small equal slices by equidistant planes parallel to the ends of the cylinder, and these slices to be condensed into heavy points arranged at equal distances along the axis, the effect produced by the cylinder will not be altered."

Dr Whewell, who seldom passes over a difficulty, has placed this assumption among his statical axioms, and expresses it in the following words. "A heavy material straight line, prism, or cylinder of uniform density may be supposed to be composed of a row of heavy points of equal weight, uniformly distributed along the line."

That this is correct with respect to a right line, no one will doubt; with respect to a cylinder it is not correct unless the line, along which the heavy points are arranged, is either the axis, or a line parallel to the axis, and limited by the vertical lines drawn through its two extremities.

Now we may conceive the slices, into which the cylinder is divided, to be so thin that they contain in their thickness only a single particle of matter, and we may therefore suppose them to be circles consisting of a number of material points equally distributed throughout their area.

The axiom assumes that each of these circles will produce the same effect by its weight as if the particles contained in it were collected at its centre.

The reason assigned for the axiom is as follows:

The weight of the whole body is equal to the weights of all the particles composing the body.

But the axiom goes beyond this, and assumes not merely what will be the magnitude of the resultant force, which is equivalent to the united influence of the weights of all the particles composing the body, *but also where that resultant will act.*

It is indeed obvious that the circle would balance round a fulcrum placed at its centre, and therefore the weights of all the particles composing the circle, may be counteracted by a single force applied at the fulcrum, and must therefore be equivalent to a single force applied at the fulcrum in the opposite direction.

But if the single force which is equivalent to the sum of the weight may be assumed to be *equal* to the sum, it would be just as correct and much shorter to apply the assumption to the cylinder itself.

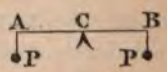
It may also be proved by experiment, that the resultant of the weights of all the particles composing any body, is always equal to their sum. For if any body be suspended by a string from a fixed point, the pressure upon that point will not be altered, however we change the shape of the body, provided we neither increase nor diminish the quantity of matter contained in the body.

Thus, if a piece of lead be suspended from one end of a balance, it will require the same weight in the other scale to counterbalance it, however its shape may be altered.

But if we suppose all the particles composing the body, to be situated in the vertical line passing through the point of suspension, the pressure upon that point will be equal to the sum of the weights, therefore it will be so in every other case.

The following proof of the proposition seems to be free from the above-mentioned objection.

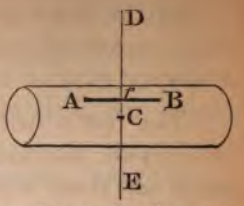
LEMMA. Let two equal weights P, P act at any two points A, B in the horizontal line AB. If a fulcrum be placed at C, the middle point of AB, P, P will balance each other, by Axiom 1, and the pressure on the fulcrum will $= 2P$ by Axiom 2.



If therefore we remove the fulcrum, a force $2P$ acting vertically upwards at C, will serve instead of a fulcrum, and balance P at A, and P at B.

But it would also balance a weight $2P$ acting at C. Therefore P at A, and P at B, can be balanced by the same force as $2P$ at C, and must therefore produce the same effect as $2P$ at C, the middle point between A and B.

Let C be the centre of the cylinder; draw DEC vertical.



Let A be any particle in the cylinder, draw $Af \perp DE$, and produce it to B, making $fB = Af$.

Then since the particles composing the cylinder are equally diffused throughout its volume, there will be at B another particle exactly equal to the particle at A.

Thus the cylinder may be divided into a number of pairs of particles equidistant from DE, and by the Lemma each pair of particles produces the same effect as if it were collected at the middle point between them, which point will always be situated in DE.

Therefore the whole of the matter composing the cylinder will produce the same effect as if it were collected in the line DE.

Therefore (Art. 8, Sect. II.) it will produce the same effect as if it were collected at C.

Prop. 2. In most treatises on Mechanics we find the converse of this proposition proved, viz. that if the weights are as the arms, they will balance, and this theorem is added as a corollary.

Hence the proof of this proposition, as it stands in the only two elementary treatises on Mechanics, now in use at this University, is scarcely as rigorous as it ought to be. The proof given in these books amounts simply to this: If P, Q be any two weights, they may be placed on a straight horizontal lever so as to balance, and when so placed they are inversely as the arms. It is true that as the cylinder AB may be of any length whatever, this proof includes an infinite number of such cases of equilibrium, but no demonstration is given that it includes the particular case which is assumed in the enunciation.

In the proof given in the text we have followed the usual custom of supposing P to be the smaller of the two weights.

The reader will observe that there is a slight difference between the enunciations of the propositions relating to the lever, and those relating to the other machines. In the propositions relating to the lever, it is required to prove that, if P and Q balance, they are to each other in a certain proportion; in the propositions relating to the other machines, it is required to prove that, if P is to Q in a certain proportion, P and Q balance.

Prop. 4. In some treatises on mechanics levers are divided into three kinds.

(1) Those in which the fulcrum is between the weight and power,

(2) Those in which the weight is between the power and the fulcrum,

(3) Those in which the power is between the weight and the fulcrum.

It will be seen that the two last kinds of lever, are comprehended in the second case that we have given. In fact, if we divide these levers, in which the weight and the power are on the same side of the fulcrum, into two classes, we might in the same way divide those levers, in which the weight and power are on different sides of the fulcrum into two classes also, according as force is gained or lost by means of them.

Numerous other examples of levers may be given.

Thus a door moved upon its hinges is an example of a lever of the second kind, the force applied by the hand being the power, the hinge being the fulcrum, and the weight of the door being the resistance which is to be overcome. This weight may be supposed to be collected at its centre of gravity, (Prop. 18) that is, at the centre of the door,

hence, force is gained when the door is opened in the usual way, and force is lost when it is opened by a force acting near the hinge.

The rudders and masts of ships, and wheelbarrows, are levers of the second kind in which force is gained, and a pair of tongs is a double lever of the second kind in which force is lost. When a ladder is raised against a wall, the end on which it rests acts as a fulcrum, and it is therefore a lever of the second kind, force is gained so long as the hands that raise it are more distant from the fulcrum than the centre of gravity (that is the centre) of the ladder, and force is lost when they are nearer.

When the arms of a lever form an angle with each other, the lever is said to be an *angular* lever. When a hammer is used for drawing a nail it is a lever of this kind; the force applied by the hand is the power, the claw is the shorter arm, and the resistance of the nail is the weight.

Prop. 6. The formula $P \cdot CM = Q \cdot CN$ comprises all the different cases of equilibrium on the lever, if we suppose CM, CN to be in all cases the perpendiculars let fall from C upon the directions of the forces P and Q.

This will be found to be by far the most convenient formula for working examples; and we may state the results proved in Props. 2, 3, 5 and 6, under the following general form.

If two forces P and Q balance on a lever, round a fulcrum C,

$$P \cdot CM = Q \cdot CN,$$

and conversely if $P \cdot CM = Q \cdot CN$, P and Q will balance round the fulcrum placed at C.

$P \cdot CM$ and $Q \cdot CN$ are called the moments of P and Q, we may increase $P \cdot CM$ by increasing either P, or CM, and we may decrease $P \cdot CM$ by decreasing either P, or

CM; but, since $Q = \frac{P \cdot CM}{CN}$, if CN remain unchanged, Q will be increased or diminished in the same proportion as $P \cdot CM$ is increased or diminished.

Hence the force Q, which P is capable of counteracting, varies as the moment of P; that is, the effort exerted by P to turn the lever round varies as the moment of P.

This is what is meant by a technical expression which is sometimes made use of, viz. that the moment of any force P, is a measure of the effort which it exerts to turn the lever round.

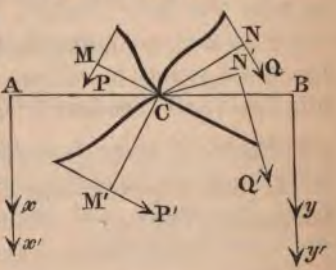
The property of the lever which we have proved for two forces is also true for any number of forces, and in its most general form may be enunciated as follows:

PROP. A. *If any number of forces* P, P′, *&c.* Q, Q′, *&c. acting upon the arms of any lever, to turn it in opposite ways, be such that,*

$$P \cdot CM + P' \cdot CM' + \ldots = Q \cdot CN + Q' \cdot CN + \ldots,$$

there will be equilibrium, and conversely, if there is equilibrium, $P \cdot CM + P' \cdot CM' + \ldots = Q \cdot CN + Q' \cdot CN + \ldots$

The proposition may be proved in the following manner.



Through C draw ACB horizontal. Let pressures x, x' act at A, perpendicularly to ACB, tending to turn the lever round in the same direction as P, P', &c., and let them be such that,

$$x \cdot CA = Q \cdot CN, \quad x' \cdot CA = Q' \cdot CN', \text{ \&c.}$$

and let pressures y, y' act at B perpendicularly to ACB, tending to turn the lever round in the same direction as Q, Q', &c., and let them be such that,

$$y \cdot CB = P \cdot CM, \quad y' \cdot CB = P' \cdot CM', \text{ \&c.};$$

$$\therefore\ x\,.\,CA + x'\,.\,CA + \ldots = Q\,.\,CN + Q'\,.\,CN' + \&c.,$$

$$y\,.\,CB + y'\,.\,CB + \ldots = P\,.\,CM + P'\,.\,CM' + \&c.,$$

the pressures x, x' ... will balance Q, Q' ..., and the pressures y, y' ... will balance P, P'...

If therefore,

$$P\,.\,CM + P'\,.\,CM' + \ldots = Q\,.\,CN + Q'\,.\,CN' + \ldots,$$

$$x\,.\,CA + x'\,.\,CA + \ldots = y\,.\,CB + y'\,.\,CB + \&c.,$$

and x, x' ... will balance y, y' ...;

$\therefore$ P, P' ... will balance Q, Q' ...

Again, if P, P' ... balance Q, Q' ...,

x, x' ... will balance y, y' ...,

and $x\,.\,CA + x'\,.\,CA + \ldots = y\,.\,CB + y'\,.\,CB + \ldots$;

$$\therefore\ P\,.\,CM + P'\,.\,CM' + \ldots = Q\,.\,CN + Q'\,.\,CN' + \ldots$$

The theorem which we have just proved might naturally be inferred from the consideration that, the moments of the forces are measures of the efforts which those forces exert to turn the lever round; if, therefore, the sums of the moments are equal, the efforts which the forces exert to turn the lever round in different directions will be equal also, and there will therefore be equilibrium.

Again, if there is equilibrium, the efforts which the forces exert to turn the lever round in different directions must be equal; and therefore the sums of the moments must also be equal.

The proportion which we have proved in the above propositions, is that which exists when there is equilibrium; if either P or Q be at all greater than is necessary to counteract the other, it will move the lever round in its own direction.

The same thing will occur in any of the other machines.

PROP. 7. A very natural objection to the truth of this proposition will suggest itself to the mind of the reader. Why, it may be asked, does a balance loaded with equal weights rest only in a horizontal position? To this objection we answer, that in this proposition the lever is supposed to be a mathematical line without weight, whereas the beam of a balance possesses both extension and weight, and does not, therefore, come under the case proved in this proposition. (Vid. note upon Prop. 23).

EXAMPLES ON THE LEVER.

1. One of the arms of a lever is 8 feet 6 inches in length; at the end of it a weight of 3qrs. is suspended; a weight of 3qrs. 18lbs. suspended at the end of the other arm preserves equilibrium. Find the length of the other arm.

Let x = the length of the other arm, the weights are 84 lbs. and 102 lbs.

Therefore by the formula $P.CM = Q.CN$,

$$102 \times x = 84 \times 8\tfrac{1}{2} \text{ feet},$$

$$102x = 714 \text{ feet}; \quad \therefore \ x = 7 \text{ feet}.$$

2. A lever 4 feet in length rests with one end on a fulcrum, a weight of 20lbs. is suspended at a distance of $1\frac{1}{2}$ feet from the fulcrum, what force acting perpendicularly at the other end will preserve equilibrium.

Let x = the force required,

$$4 \times x = 20 \times 1\tfrac{1}{2},$$

$$4x = 20 \times \tfrac{3}{2} = 30,$$

$$x = 7\tfrac{1}{2} \text{lbs}.$$

3. A lever AB supported on props at A and B, is divided in C so that $AC : CB :: 5 : 2$; a half hundredweight is suspended at C, find the portion supported by each prop.

The case will be the same as if two forces acted perpendicularly at A and B round a fulcrum placed at C, the pressure on the fulcrum being equal to a half hundredweight.

Let x = force exerted at A; $\therefore 56 - x$ = force exerted at B, (Prop. 2),

$$\text{and } x.AC = (56 - x)\,CB;$$

$$\text{or, } x.\frac{AC}{CB} = 56 - x;$$

$$\text{but } AC : CB :: 5 : 2; \quad \therefore \frac{AC}{CB} = \frac{5}{2};$$

$$\text{hence } x \times \tfrac{5}{2} = 56 - x,$$

$$5x = 112 - 2x,$$

$$7x = 112,$$

$$x = 16 \text{ lbs.} = \text{weight supported at } A,$$

$$\text{and } 56 - x = 40 \text{ lbs.} = \text{weight supported at } B.$$

Note.—In the following Examples the forces are supposed to act perpendicularly on the lever, unless the contrary be asserted.

4. At the ends of a lever 6 feet in length are suspended two weights which are to each other as 5 : 2, where must the fulcrum be placed in order to preserve equilibrium.

Let x = the length of one arm; $\therefore 6 - x$ = the length of the other,

by Prop. 2, $x : 6 - x :: 5 : 2$; $\therefore 2x = 30 - 5x$;

$$\therefore\ 7x = 30\ ;\ \therefore\ x = \frac{30}{7} = 4\frac{2}{7},\ \text{and}\ 6 - x = 6 - \frac{30}{7} = 1\frac{5}{7}.$$

Therefore the fulcrum must be placed at a distance of $4\frac{2}{7}$ feet from the smaller weight, and of $1\frac{5}{7}$ feet from the larger.

5. The arms of a lever are as 2 to 3, the pressure on the fulcrum is 4lbs. Find the weights, the lever being one of the second class.

Let x = the smaller weight; therefore, by Prop. 3,

$x + 4$ = the larger weight, and $x : x + 4 :: 2 : 3$.

Hence $3x = 2x + 8\ ;\ \therefore\ x = 8\ ;$

therefore the weights are 8 and 12 lbs.

6. A capstan bar has six spokes, of which two are 4 feet long, and four 3 feet long, at the extremities of each of these spokes, a force of $\frac{1}{2}$ cwt. acts perpendicularly to turn the lever round in the same direction: a force acting perpendicularly at the end of a spoke 32 feet long preserves equilibrium. What must be the magnitude of the force?

Let x = the force,

Then $32 \times x = (4 + 4 + 3 + 3 + 3 + 3)\ \frac{1}{2}$ cwt. by Prop. A.

$= 20 . \times \frac{1}{2}$ cwt. $= 10$ cwt. $= 1120$ lbs.

$\therefore\ x = 35$ lbs.

7. Given the apparent weights in a balance of which the arms are unequal, to find the true weight.

Let a, b be the apparent weights, x the true weight, m, n the lengths of the arms.

Now x, acting at an arm m balances a, acting at an arm n, and x acting at an arm n balances b, acting at an arm m.

$$\therefore\ mx = an, \qquad nx = bm;$$

$$\therefore\ mnx^2 = abmn, \quad x^2 = ab, \text{ and } x = \sqrt{ab}.$$

8. The arms of a balance being equal, but one scale loaded, find the true weight from the apparent weights.

Let w = the weight with which the false scale is loaded, and let a, b be the apparent weights, x the true weight.

Then when the body is put into the loaded scale,

$$x + w = a,$$

when it is put into the other scale,

$$x = b + w.$$

$$\therefore\ 2x + w = a + b + w, \quad \text{and } 2x = a + b;$$

$$\therefore\ x = \frac{a + b}{2}.$$

9. The arms of a lever are three and four feet, and a weight of 6 lbs. is suspended at the end of the longer arm. What weight must be suspended at the end of the other arm to preserve equilibrium ?

10. At a distance of two feet from the fulcrum of a lever is suspended a weight of 12 lbs. Where must a weight of 8 lbs. be suspended in order to preserve equilibrium ?

11. The arms of a lever are as 5 : 6. The pressure on the fulcrum is 11 lbs., the lever being of the first kind. Find the weights, when there is equilibrium.

12. If the lever were of the second kind, what would the weights be?

13. The length of a lever of the first kind is 12 feet, and the weights are 2 and 4lbs. What must be the lengths of the arms in order that there may be equilibrium.

14. What would be the lengths of the arms, if the lever were of the second kind?

15. In a lever of the second kind the arms are as 5 : 4; a weight of 30lbs. is suspended at the end of the longer arm. What power must be applied at the end of the other arm to preserve equilibrium, and what will be the pressure on the fulcrum?

16. If the weights are as 6 : 4, and the whole length of the lever is 9 feet; what will be the length of the shorter arm?

17. A straight horizontal lever rests with one end on a prop; at a distance of 2 feet from the prop, a weight is suspended, and a force acting perpendicularly on the lever at a distance of 6 feet from the prop, preserves equilibrium. If the prop sustains a pressure of 4lbs., what will be the weight supported, and the magnitude of the force which counterbalances it?

18. Two forces which are to each other as 3 : 2, act perpendicularly on a lever of the second kind, at a distance of a foot from each other. Find the length of the arms when there is equilibrium.

19. Two men of the same height bear a weight hung on a pole which rests on their shoulders. Where must the weight be placed in order that one may support twice as much of the load as the other?

20. The arms of a bent lever are inclined at the same angle to a horizontal line passing through the fulcrum. The arms are to each other as 3 : 2, and a weight of 24 lbs., is suspended at the end of the longer arm; what weight must be suspended at the end of the other arm to preserve equilibrium?

21. In a false balance, whose arms are unequal, the apparent weights are 4, and 9 pounds. Find the true weight.

22. A horizontal lever, 12 feet in length, rests with one end on a prop; at distances of 4 and 6 feet from the prop are suspended weights of 20, and 32 lbs.; a force of 2 cwt. acts perpendicularly upwards on the lever at the distance of a foot from the other end. What force will be required at that end to preserve equilibrium?

23. If the weights 1, 2, 3 lbs., be suspended at the distances of 6, 12, and 18 inches from the fulcrum on one arm of a straight lever, and 2, 3, 4 lbs. be placed at the distances of 4, 10, 12 inches from the fulcrum on the other arm; find where a weight of 1 lb. must be placed in order to preserve equilibrium.

Prop. 8. In order to understand this proposition, it is necessary to have a clear conception of the following step.

If a force act upon a lever, and the line of the force's action pass through the fulcrum, we may suppose the force to be applied immediately at the fulcrum. The tendency of the force, therefore, will be to move the fulcrum; and since the fulcrum is supposed to be a fixed point capable of counteracting any pressure that may be applied to it, this tendency will be entirely counteracted by the resistance of the fulcrum, and the lever will remain at rest. But if the line of the force's action does not pass through the fulcrum,

the resistance of the fulcrum will not counteract the force, and it will move the lever round the fulcrum.

In the proof of this proposition it is shewn that the line of action of the resultant must pass through D; now a force in the direction either of AD or DA, would satisfy this condition; but it is obvious that the resultant of AB, AC cannot act in the direction DA, and therefore it must act in the direction AD.

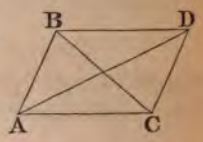
This theorem is usually called "The Parallelogram of Forces," and admits of a rigorous analytical proof, independent of the property of the lever. Most writers make this the fundamental proposition of Statics, and deduce the property of the lever from it; but their demonstrations of this proposition require the aid of mathematical reasoning of a higher kind than can be expected from an ordinary reader.

On this point the student may consult Pratt's Mechanics, Earnshaw's Statics, or Lagrange's Mechanique Analytique.

If two forces act in the same right line, it is clear that their resultant will be the sum of the forces when they act in the same direction, and the difference of the forces when they act in opposite directions. Thus if two forces of 6, and 8 lbs. act in the same right line, their resultant will be a force of 14 lbs. when they act in the same direction, and of 2 lbs. when they act in opposite directions. It is obvious also that in the latter case, the resultant will always act in the direction of the larger force.

Cor. From this proposition it follows that, if any force be represented in magnitude and direction by a right line, and a parallelogram be formed of which this line is the diagonal; the two forces represented in magnitude and direction by the adjacent sides of the parallelogram will be together equivalent to the original force.

It is necessary that the reader should clearly understand which of the diagonals of a parallelogram will represent the resultant of the two forces represented by the adjacent sides.



The resultant of AB and AC, or AB and BD, is AD;

of AB and CA, or AB and DB, is CB;

of BA and CA, or BA and DB, is DA;

of BA and AC, or BA and BD, is BC.

The reader must not confound the direction of a force with its line of action. The diagonals represent the resultants in magnitude and direction, and not necessarily in line of action.

Prop. 9. From this proposition we see that, if three forces can keep a point at rest, they may be represented by the sides of some triangle, hence if there are three forces such, that the lines representing them cannot form a triangle, they cannot keep a point at rest. Thus forces represented by 3, 8, and 20, could not keep a point at rest, for since any two sides of a triangle must be greater than the third side; the lines representing these forces could not form a triangle. If, however, the sum of two of the forces were *equal* to the third, the first two forces would balance the third force, and keep the point at rest, provided we suppose the three forces to act in the same straight line, the third force acting in a direction opposite to that of the other two.

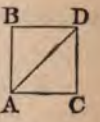
If the reader will examine Euclid I. 22, he will see that this is the limiting case of the problem, and answers

to the construction which occurs when the two circles touch instead of cutting each other.

EXAMPLES.

1. Two forces of 8 lbs. each act on a point at right angles to each other. Find the magnitude and direction of their resultant.

Let AB, AC represent the forces. Complete the parallelogram, and draw the diagonal AD;



AD will represent the resultant,

and $AD^2 = AC^2 + CD^2$ (since $\angle C$ = right angle by Cor. 1. 46)

$= AC^2 + AB^2$ (1. 34) $= 64 + 64 = 128$;

$\therefore AD = \sqrt{128} = 8\sqrt{2}$ which determines the magnitude of the resultant.

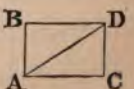
Again $CD = AB = AC$; $\therefore \angle CDA = \angle DAC$,

and C = right angle, $\therefore$ each of the angles CDA, $DAC = \frac{1}{2}$ a right angle. (1. 32.)

Therefore $DAC = \frac{1}{2}$ a right angle, which determines the direction of the resultant.

It is manifest that the direction of the resultant bisects the angle BAC, from the consideration that the component forces are equal, and that the resultant therefore must be equally inclined to each of them.

2. Shew that if the components are represented by 3 and 4, and the resultant by 5, the components act at right angles to each other.



Complete the parallelogram as before; and join AD.

Then $AB = 3$, $AC = 4$, and $AD = 5$,

also, $DC = AB = 3$.

$\therefore\ AC^2 + DC^2 = 4^2 + 3^2 = 16 + 9 = 25 = 5^2$,

$= AD^2$.

$\therefore$ (I. 48) the $\angle C$ = right angle, hence by Cor. I. 46,

$\angle BAC$ = right angle, or the components act at right angles to each other.

3. The component forces act at right angles to each other, one of them being 15 lbs. If the resultant force be 25 lbs., what will be the magnitude of the other component?

The same construction being made as in the last example,

$$AD^2 = AC^2 + CD^2 = AB^2 + AC^2,$$

$$\therefore\ AC^2 = AD^2 - AB^2 = 25^2 - 15^2 = 625 - 225 = 400.$$

$\therefore\ AC = 20$, or the magnitude of the other component will be 20 lbs.

4. Find a point in any triangle such that if 3 right lines be drawn from it to the angular points of the triangle, the forces represented by the three right lines will keep the point at rest.

Let o be the point required, join oA, oB, oC. oA counteracts oB, oC, and must therefore be equal and opposite to their resultant.

Hence, if in Ao produced we take $oH = oA$, oH will be the resultant of oB, oC.

Therefore if we join BH, HC, $oBHC$ will be a parallelogram;

produce Bo, Co to meet AC, AB, in e and d.

Then ef is parallel to FC and ce is parallel to CF.

Hence $Ae : eC = AF : FB = 1 : 1$.

and $AF = FB$. $\quad Ae = eC$.

$\therefore$ AC is bisected in e. In the same way we may shew that AB is bisected in f.

Hence, if we bisect AB, AC in f and e and join Ce, Bf; o, the point of intersection of Ce, Bf, will be the point required.

5. Two forces of 7 and 5 pounds act upon a point in the same direction. What will be the magnitude and direction of the resultant?

6. What will be the magnitude and direction of the resultant, when the forces act in opposite directions?

7. Can three forces represented in magnitude by 9, 6, 4, keep a point at rest?

8. Can three forces of 9, 6, and 1 lb. keep a point at rest?

9. Two forces of 9 and 12 lbs., act at right angles to each other. Find the magnitude of their resultant.

10. Two forces of 21 lbs. and 28 lbs. have a resultant of 35 lbs. Shew that the two component forces act at right angles to each other.

11. Two forces, which act at right angles to each other, have a resultant of $1\frac{1}{4}$ lb. One of the component forces is 1 lb. Find the magnitude of the other.

12. In pulling a weight along the ground by a rope inclined to the horizon at half a right angle, I exerted a power of 40 lbs. Required the force with which I draw the body horizontally.

Prop. 10. If we wish to raise a weight by means of a lever, the fulcrum must be placed a little above that end of the lever on which the weight rests. The lever is then turned round the fulcrum until the end at which the power is applied is a little lower than the other end. If we wish to raise the weight still higher, we must support the weight until the fulcrum is again raised above it, when the machine can be put into operation again.

The wheel and axle, as we have shewn already, is merely another form of the lever, but it possesses one great advantage over the common form of the lever, viz. that its action is continuous. As long as we turn the wheel round the string will continue to coil itself round the axle, and the weight will rise.

A contrivance called a ratchet wheel is often annexed to this machine, by means of which the wheel is rendered capable of moving round only in one direction, so that if we remove the power, the rope will not uncoil itself, and the weight therefore will remain fixed.

In the forms of the wheel and axle, which are in common use, a power acting by means of a spoke, generally supplies the place of the smaller weight.

One of the commonest forms of this machine, is the contrivance used for drawing water from a well; in this contrivance, which is generally called the windlass, instead of the whole circumference of the wheel, we have merely a single spoke of it, which is furnished with a handle, and called a winch.

The treadmill furnishes us with another instance of the wheel and axle, in which the wheel is turned by the weight of men placed at is circumference, who ascend as fast as the wheel revolves, so as to maintain their position continually at the extremity of the horizontal diameter.

PROP. 12. The most advantageous and perfect form of this kind of pulley, is that which, after the name of its inventor, is called White's Pulley.

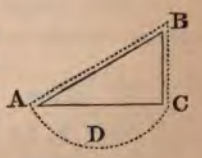
In the common form of this kind of pulley, the wheels revolve with different degrees of velocity, so that the friction and wear of the different pulleys is unequal. This defect Mr White remedied by making the wheels of different magnitudes, such that they all revolve in the same time.

In addition to this alteration, instead of using a number of different wheels, he cut a number of circular grooves upon the same solid wheel, and thus instead of the multiplied friction of several distinct axles and blocks, there is in White's Pulley only the friction of a single axle, and a single block.

This pulley would be exceedingly useful, if it could be constructed with perfect accuracy ; but great difficulty is found in giving to the grooves the true proportion; and hence the cord slides in the grooves, and other inconveniences arise which greatly circumscribe its practical utility.

PROP. 14. The property of the inclined plane has been proved independently by Stevin.

The substance of the proof is as follows.



If we suppose a uniform and perfectly flexible endless rope or chain to be passed round the inclined plane *ABC*, so that the upper part rests on the plane, and the lower part hangs in a festoon *ADC*;—the chain will remain at rest. Now the two portions *AD*, *DC* of the festoon must needs be exactly similar,

and exert tensions in opposite directions; therefore they are in equilibrio of themselves;

$\therefore$ the parts AB, BC must counteract each other,

and $P : W ::$ weight of part BC : weight of part AB,

$::$ BC : AB, since the chain is uniform.

This proof will be found given at full length in the introductory chapter of Lagrange's *Mechanique Analytique*.

This proof, which is one of the simplest and most ingenious to be found in the whole range of Mathematics, would be perfect but for the assumption that the chain will necessarily remain at rest. It is true that if the chain once begins to move, the motion will continue for ever, because the chain will always retain the same form, and that a perpetual motion is contrary to all our received ideas. But the cessation of motion that we constantly observe in all things around us, is attributable to forces of resistance, such as friction, the resistance of the air, &c. Without these forces of resistance a pendulum once put into motion, would oscillate for ever, and as in the case of the chain all such forces are neglected, it may be questioned whether the assumption that the chain will rest, instead of moving round the plane for ever, be so clear and self-evident as to afford no ground of objection to a captious reader.

FRICTION.

The proportions which have been proved above are those which exist when there is equilibrium.

In theory, if either P or Q be at all greater than is necessary to keep the other in equilibrio, it will move the machine round in its own direction. In practice, however, we find that either the power or the weight is susceptible

of a certain amount of increase or diminution without affecting the equilibrium.

For instance, if we form an inclined plane so that, theoretically, a weight of 20lbs. will be balanced by a power of 4lbs.; the equilibrium will not be disturbed, although we add a few pounds to the weight, or take a few ounces from the power. This increase or diminution however must not exceed certain limits; which are different according to the form of the body, and the nature of the substances employed.

This resistance to motion, is supposed to be caused by the inequalities of the surfaces which are in contact, and is called friction.

The effect produced by the friction of two surfaces in contact, is very nearly analogous to the resistance to motion which is offered by the hairs of two brushes placed one upon the other.

Friction is diminished by making the surfaces in contact smoother, whether this be done by polishing the surfaces, or greasing them; if however the surfaces receive more than a certain degree of polish, they will stick together or cohere, and the resistance to motion will be increased. For instance, when two pieces of highly polished plate glass are placed in contact, they sometimes stick together so closely, owing to the force of cohesion, that they cannot be separated without a fracture.

ON COMBINATIONS OF MACHINES.

We have already stated the disadvantages of the lever as a contrivance for raising bodies, nor is the wheel and axle without its own peculiar defects.

For the mechanical advantage of this, as well as of all other machines, consists in our being able by means of it to counteract a large force by means of a small one, and may be measured by the ratio of $W : P$, that is, by the fraction $\frac{W}{P}$.

In the wheel and axle $\frac{W}{P} = \frac{\text{radius of wheel}}{\text{radius of axle}}$, the mechanical advantage may therefore be increased, either by diminishing the radius of the axle, or by increasing that of the wheel.

In practice, however, we cannot diminish the thickness of the axle, beyond a certain limit, for if we do so, it will not be strong enough to support the pressure upon it, and will break.

Neither can we increase the size of the wheel beyond a certain limit, for if we do so, the machine will occupy so great a space, and be so inconvenient to turn round, that it will be utterly useless, as the reader will see, if he consider the common windlass for raising water from a well.

If therefore we wish to raise a large weight by means of a small power, we must have recourse to some combination of the different mechanical powers. In all such combinations the pressure exerted by one machine, instead of supporting a weight, acts as a power upon the next machine, and the mechanical advantage of the compound machine may obviously be found by multiplying together the mechanical advantages of all the simple machines which compose it. If we disregard the friction, any power which is at all greater than that which would produce equilibrium, will put the machine into motion; if friction be taken into account, the power must be sufficient to overcome the friction as well as to balance the weight.

EXAMPLES.

1. On a wheel and axle, a power of 10oz. balances a weight of 10lbs. Find the ratio of the radii.

radius of axle : radius of wheel :: power : weight,

:: 10oz. : 10lbs.

:: 10 : 160,

:: 1 : 16.

2. If the radius of the axle be 2 inches, and that of the wheel 3 feet, what power will be required to support a weight of 48lbs.?

The power : 48lbs. :: 3 inches : 2 feet,

3 : 24;

$$\therefore \text{ the power } = \frac{3 \times 48}{24} \text{lbs.} = 6 \text{lbs.}$$

3. In a system of pullies where the same string passes round each pulley,

$$P : W :: 1 : 8.$$

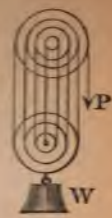
Find the number of pullies.

$P : W :: 1$: the number of strings at the lower block;

therefore there must be eight strings at the lower block, and there will be 4 pullies.

4. In a similar system of pullies, there are 3 pullies at the lower block, and the end of the string is fastened to the lower block.

Find the ratio of the power to the weight.



In this case there will be one more string at the lower block than 2 × the number of pullies at the lower block;

$$\therefore\ P : W :: 1 : 7.$$

5. In a system of pullies where each pulley hangs by a separate string, the number of pullies is 5. What weight will a power of 5 lbs. support?

$P : W :: 1 : 2$ raised to that power whose index is the number of moveable pullies,

$$:: 1 : 2^5,$$

$$\therefore\ W = 2^5 \times P = 32 \times 5 \text{ lbs.} = 160 \text{ lbs.}$$

6. In a similar system of pullies $P : W :: 1 : 64$. Find the number of moveable pullies.

Here $64 = 2$ raised to that power whose index is the number of moveable pullies.

$$= 2^6,$$

$\therefore$ the number of moveable pullies is 6.

7. In an inclined plane the height is 4 feet, and the length is 48 feet. What power is required to balance a weight of 3 cwt.?

Since $P : W ::$ height of plane : length of plane,

$$P : 3 \text{ cwt.} :: 4 \text{ feet} : 48 \text{ feet};$$

$$\therefore\ P = \frac{4 \times 3}{48} \text{ cwt.} = \tfrac{1}{4} \text{ cwt.} = 28 \text{ lbs.}$$

8. In a combination of three levers the arms are as 3 : 2, 5 : 2, and 7 : 1.

Find the mechanical advantage of the combination.

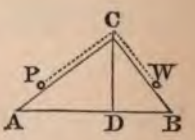
The mechanical advantages of the separate levers are

$$\frac{3}{2}, \frac{5}{2}, \text{ and } \frac{7}{1},$$

$\therefore$ the mechanical advantage of the combination

$$= \frac{3}{2} \times \frac{5}{2} \times \frac{7}{1} = \frac{105}{4}.$$

9. If two weights P and W support each other on a double inclined plane ACB, find the ratio between P and W.



Draw $CD \perp AB$.

Since there is equilibrium the tension of the string is the same throughout, therefore P would be sustained by the same weight on the single inclined plane ACD, that would sustain W on the single inclined plane BCD. Let this weight be W'.

$$P : W' :: AC : CD \text{ or } \frac{P}{W'} = \frac{AC}{CD},$$

$$W' : W :: CD : CB \text{ or } \frac{W'}{W} = \frac{CD}{CB};$$

$$\therefore \frac{P}{W'} \times \frac{W'}{W} = \frac{AC}{CD} \times \frac{CD}{CB}, \quad \therefore \frac{P}{W} = \frac{AC}{CB},$$

$\therefore P : W :: AC : CB$, or the weights are proportional to the sides of the plane.

10. In a combination of the wheel and axle, with the first system of pullies, the radius of the wheel is six times as great as that of the axle, and there are 5 pullies

at the lower block. What weight will a power of 3 lbs. support?

$$\frac{W}{P} = \frac{6}{1} \times \frac{2 \times 5}{1} = 60; \quad \therefore \quad W = 60 \,.\, P,$$

and $W = 180$ lbs.

11. In a combination of the wheel and axle, the second system of pullies, and the inclined plane, height of plane : its length :: 3 : 7, there are 4 moveable pullies, and radius of axle : radius of wheel :: 7 : 20. What power will be required to support a weight of 640 lbs.?

$$\text{Here } \frac{W}{P} = \frac{20}{7} \times \frac{7}{3} \times \frac{2^4}{1}$$

$$= \frac{20 \,.\, 7 \,.\, 16}{7 \,.\, 3} = \frac{320}{3};$$

$$3\,W = 320 \,.\, P;$$

$$\text{or, } P = \frac{3 \,.\, W}{320} = \frac{3 \,.\, 640}{320} \text{ lbs.} = 6 \text{ lbs.}$$

12. What force will be required to support a weight of 960 lbs. on the wheel and axle, if the ratio of the radii is 8 : 1?

13. Under the same circumstances, what weight will a power of 6 lbs. support?

14. If a power of 35 lbs. support a weight of 3 qrs. on a wheel and axle, and the radius of the axle be 5 inches, what must be the radius of the wheel?

15. In a system of pullies of the first kind, there are 7 pullies at the lower block. What weight will a power of 2 lbs. support?

16. In a similar system, the number of pullies is 6. What power will support a weight of 3 cwt.?

17. If one end of the string be fastened to the lower block, and a power of 6 lbs. supports a weight of 42 lbs. what is the number of pullies at the lower block?

18. In the second system of pullies, if the number of pullies be 7, what weight will a power of 3 lbs. support?

19. If the number of moveable pullies be 5, what power is required to sustain a weight of 2 cwt.?

20. If a power of 14 lbs. supports a weight of 8 cwt., what is the number of moveable pullies?

21. A weight of 5 lbs. is supported on an inclined plane by a power of 3 lbs. The height of the plane is 6 inches. Find its length.

What will be the pressure upon the plane?

22. The height of an inclined plane is to its length as 12 : 21. What power will be required to support a weight of 1 cwt.?

23. Under the same circumstances, what weight will a power of 1 cwt. support?

24. In a combination of a wheel and axle and an inclined plane, radius of axle : radius of wheel :: 1 : 7 and height of plane : its length :: 14 : 36. What power will support a weight of 3 cwt.?

25. In a combination of 4 levers, the ratios of the arms are 5 : $1\frac{1}{2}$, 7 : 2, 11 : 3, and 54 : 7. What weight will a power of 5 lbs. support?

26. Two weights of 6 and 7 lbs. support each other on a double inclined plane. If one of the sides be 3 feet, what will be the length of the other side?

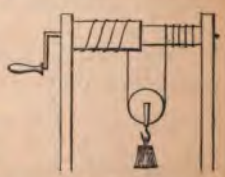
27. In a combination of two wheels and axles, and a system of pullies of the second kind, the number of pullies is 3, the radii of the wheels are 3 and 2 feet, and those of the axles 5 and 6 inches. Find the mechanical advantage of the combination.

One great source of imperfection in all systems of pullies is the friction, in addition to which, it must be recollected, that in the formulæ which we have proved in Props. 12 and 13, the weights of the pullies themselves have not been taken into account.

In both the systems of pullies which are described in the text, the weights of the pullies act in opposition to the power, and thus diminish the efficacy of the machines. It is however, possible to construct a system of pullies so that the weights of the pullies themselves shall act in opposition to the resistance that is to be overcome, and thus increase the efficacy of the machine. The mechanical advantages of all such combinations may be calculated upon the same general principles as we have employed in Props. 12 and 13.

We now proceed to describe a peculiarly simple and effective machine, called the differential axle, the invention of which is generally ascribed to Mr Eckhardt, although, like many other modern discoveries, it appears to have been known to the Chinese from an early period.

In the combination represented by the annexed figure, the weight is fastened to a pulley, the strings of which are coiled round two axles of unequal radii, and tend to turn the machine round in different directions. The axles are firmly fixed to each other, and have a common axis. As the tensions of the two strings are equal (Prop. xi.), that which is attached to the larger axle would, under these circumstances preponderate, and turn the machine round in its own direction; equilibrium is therefore preserved by a power,



which acts by means of a winch, and tends to turn the machine round in the opposite direction.

Let r, r' be the radii of the axle, R the length of the winch, P the power, and W the weight.

The tension of each string will be $\frac{W}{2}$,

Hence, if P act perpendicularly on the winch,

$$P.R + \frac{W}{2}r' = \frac{W}{2}r, \quad \therefore \; P.R = \frac{W}{2}(r - r');$$

$$\text{and } \frac{W}{P} = \frac{2R}{r - r'}.$$

The mechanical advantage, therefore, of this machine may be increased to any extent, by diminishing $r - r'$, that is, by making the radii of the axles more nearly equal, without increasing the leverage of the power so as to make the machine unwieldy, or diminishing the thickness of the axles, so as to render them too weak to support the weight. Almost the only disadvantage attending this machine is, that a very great length of cord is required to raise the weight through a comparatively small height.

The principle proved in the fifteenth and sixteenth propositions may be more briefly stated as follows.

If P and W balance on any one of the simple machines, and the whole be put into motion,

$$\frac{P}{W} = \frac{P\text{'s velocity in the direction of its action}}{W\text{'s velocity in the direction of its action}}.$$

W always acts vertically, and in both systems of pullies, and the wheel and axle, W, if it moves at all, moves vertically, that is, in the direction of its action; hence its velocity in the direction of its action will be the

same as its actual velocity. But in the inclined plane, the weight, if it move at all, must move, not in the direction of its action, that is, vertically, but along the plane, in the direction of the power. Hence, its velocity in the direction of gravity will not be the same as its actual velocity, and the theorem, as stated in the case of the pullies and the wheel and axle, will not be true for the inclined plane.

It will be seen that the theorem as we have stated it, is applicable to the inclined plane as well as the other machines.

In the case of any combination of machines, the ratio of the power's velocity in the direction of its action to the weight's velocity in the direction of its action; may be found by multiplying the ratios which obtain in each of the simple machines of which the compound machine is composed.

Now the ratio of the power to the weight in the compound machine may also be found by multiplying the ratios which obtain in each of the simple machines, and since

$$\text{on each of these } \frac{P}{W} = \frac{P\text{'s velocity, \&c. \&c.}}{W\text{'s velocity, \&c. \&c.}},$$

the compounded ratios will also be equal.

In all combinations of machines therefore,

P : W :: P's velocity in the direction of its action : W's velocity in the direction of its action.

This principle is called the *principle of virtual velocities*, and is made the fundamental proposition of Statics by Lagrange, and some other writers, who enunciate the principle under a form far more general than we have given above, and furnish us with a complex but rigorous analytical demonstration.

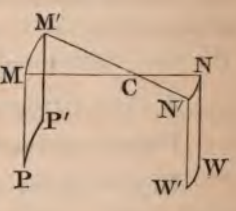
Although the principle is not applied to the lever in the schedule, it is applicable to the lever as well as to the other machines, as might naturally be expected from the consideration that the lever and the wheel and axle are merely different forms of the same machine, and therefore that any general principle that applies to the one form, will be equally applicable to the other.

We proceed to prove this proposition as applied to the straight lever, when the forces act perpendicularly upon it, and the reader will see that the proof is substantially the same as that given in Prop. 20.

If P *and* W *acting perpendicularly on a straight lever balance, and the system be put into motion,*

$$P \times P\text{'s velocity} = W \times W\text{'s velocity.}$$

Let the original position of the lever be MN, and let it be turned round into the position $M'N'$; M, N will evidently describe circular arcs round the centre C, and since the strings MP, NW always remain vertical, P and W will describe arcs similar and equal to MM' NN'.



Now the angles $M'CM$, NCN', subtended by MM' NN' at the centre C are equal. (v. 15.)

Therefore by the Lemma of Prop. xv.

$$MM' : NN' :: CM : CN.$$

But $P : W :: CN : CM$, since there was originally equilibrium,

$:: NN' : MM'$,

$:: WW' : PP'$,

:: space described by P : space described by W in the same time,

:: velocity of W : velocity of P,

or P. P's velocity = W. W's velocity.

The principle of virtual velocities as applied to machines, is often stated under the following form.

What is gained in power, is lost in time.

This does not mean that the space described by the weight is really described in a longer time than that described by the power. These spaces are necessarily described in the same time. But it takes a longer time to move the weight through any given space, than it would take to move the power through the same space, and the times required to move the weight and power through this space bear to each other the same ratio that the weight and power do.

In proportion therefore as we gain in power, (that is, in proportion as the power required to support the weight is less than the weight,) in the same proportion (the time required to move the weight through any given space, is greater than the time required to move the power through the same space, and therefore) we gain in time.

A more obvious statement of this principle would be,

What is gained in power is lost in velocity.

In the theoretical view which we have just taken of the matter, it is supposed that the power, which balances the weight, is sufficient to move it; in practice however a considerably greater force than this is required to put a machine into motion, owing to the resistance of the air, and of friction, so that in reality we do not gain so much in power as we lose in time. In fact, the total amount of force expended in moving a body through a given space is greater when we use a machine, than it would be if we could apply a sufficient amount of force immediately.

What then, it may be asked, do we gain by the use of machines? To this we answer, that

(1) We are able to change the direction of the moving power, so that it can be applied more conveniently than it otherwise could, as is the case in the fixed pulley.

(2) We are able to impart a great velocity to a small body, by means of a considerable force, as in the knife-grinder's wheel.

(3) We are able to move a great weight, by means of a small power, as is the case in the wheel and axle.

When we say that in a machine the power sustains the weight, we use a popular but incorrect expression, in reality the power supports only such a portion of the weight as is equal to its own force, the rest of the weight is distributed among, and supported by, the fixed points or props of the machine.

Although, however, in a state of equilibrium, the weight is not supported by the power, yet if the machine be put into motion, the weight is actually raised by means of the power; in such cases the machine enables us as it were to accumulate and condense the power which we employ.

For instance, it requires a certain amount of force to raise a hundred-weight through a height of 20 inches, and it would clearly require twenty times as great an effort to raise a ton through the same distance; but it would obviously be impossible for any man to exert so great an effort instantaneously; he might however divide the ton into twenty cwts., and carry each cwt. separately, in which case he would still exert twenty times the effort that he exerted in carrying the single cwt. But if the substance were of such a nature that he could not divide it, he would be utterly unable of himself to perform the task at all.

He might however construct a machine by means of which he could (if we suppose the machine to be free from all resisting forces) raise the ton through one inch by the same effort, as that, by which he could raise the single cwt. through 20 inches, and consequently he could raise the whole ton through 20 inches by exerting twenty times this effort.

In practice, however, owing to the forces of resistance, it would require a greater effort to raise the ton through one inch by means of a machine than it would to raise a cwt. through 20 inches without a machine, and it would therefore require more than twenty times this effort to raise the ton through 20 inches.

The total amount, therefore, of force which is required to raise the ton through 20 inches, is greater when a machine is used than it would be without it, provided a force of sufficient intensity could be applied immediately; although as a man could not of himself apply at once, a force of sufficient intensity, he could not raise the ton at all without the help of a machine.

It is often possible to apply a small force for a long time where we cannot apply a large force for a short time, and the advantage of most machines consists in their enabling us to produce the same effect, by means of a small force applied for a long time, that would be produced by a great force applied for a short time.

This is the case in all the mechanical powers, where the machines enable us to condense, as it were, or concentrate a number of small efforts into a single great one.

Sometimes, however, it is necessary to produce a great velocity in the body that is to be moved, by means of a power which possesses a small velocity; this, which is the converse of the former case, may be effected by interchanging the situations of the weight and power, and it is obvious that in all such cases, the power will act at a mechanical disadvantage, and must be greater than the weight.

A turning-lathe is a machine of this kind, in which velocity is gained at the expense of power.

EXAMPLES.

In the following examples, the motion is supposed to be uniform.

1. Compare the velocities of two bodies, one of which passes through 320 feet in a minute, and the other through $3\frac{1}{2}$ yards in $1\frac{1}{4}$ of a second.

sec. min. yards.

$1\frac{1}{4}$: 1 :: $3\frac{1}{2}$: the space which the second body would describe in a minute.

$\therefore$ this space $= 60 \times 3\frac{1}{2} \div 1\frac{1}{4}$ yards,

$$= 60 \times \frac{7}{2} \times \frac{4}{5} \text{ yards} = 168 \text{ yards},$$

$$= 504 \text{ feet},$$

and velocity of the first body : velocity of the second body

:: space described by the first body in a minute : space described by the second body in the same time,

:: 320 : 504

:: 40 : 63.

2. If one body moves uniformly through 5 feet in $\frac{1}{2}$ a second, and another moves uniformly through 100 yards in a minute, what will be the ratio of their velocities?

The first body describes 5 feet in $\frac{1}{2}$ a second, and therefore will describe 600 feet in a minute.

The second body describes 100 yards or 300 feet in the same time;

$\therefore$ velocity of first body : velocity of second body

:: 600 feet : 300 feet

:: 2 : 1.

3. If, in a combination of three wheels and axles, the velocity of the weight is 48 times smaller than that of the

power, what will be the mechanical advantage of the combination?

$$\text{Since } \frac{W}{P} = \frac{P\text{'s velocity}}{W\text{'s velocity}} = \frac{48}{1};$$

$\therefore$ the ratio 48 : 1 represents the mechanical advantage.

4. If in a combination of levers the mechanical advantage is represented by the ratio 29 : 2, and the weight moves with a velocity of 2 inches in a second, what will be the velocity of the power?

$$\frac{\text{velocity of power}}{\text{velocity of weight}} = \frac{W}{P} = \frac{29}{2};$$

$$\therefore \text{ velocity of power } = \frac{29}{2} \text{velocity of weight}$$

$$= \frac{29}{2} \times 2 \text{ inches} = 29 \text{ inches};$$

or the power moves with a velocity of 29 inches in a second.

5. If in a wheel and axle the mechanical advantage is represented by the ratio 21 : 5, and the weight moves through a space of 10 inches in $1\frac{3}{4}$ second, what space will the power describe in a minute?

$$\frac{\text{velocity of power}}{\text{velocity of weight}} = \frac{W}{P} = \frac{21}{5};$$

and $1\frac{3}{4}$: 60 :: 10 inches : the number of inches described by W in a minute;

$$\therefore \text{ number of inches described by } W \text{ in a minute} = 60 \times 10 \times \frac{4}{7},$$

$$\text{and } \frac{\text{space described by } P \text{ in a minute}}{\text{space described by } W \text{ in a minute}} = \frac{P\text{'s velocity}}{W\text{'s velocity}} = \frac{21}{5};$$

$\therefore$ space described by P in a minute $= \frac{21}{5} \times$ space described by W in a minute $= \frac{21}{5} \times 60 \times 10 \times \frac{4}{7}$ inches $= 1440$ inches $= 40$ yards.

6. Compare the velocities of two bodies which move through 6 and 8 feet in the same time.

7. Compare the velocities of two bodies, of which one moves through $3\frac{3}{4}$ feet in a second, and the other through $2\frac{1}{2}$ feet in the same time.

8. Compare the velocities of two bodies, of which one moves through 4 feet in 2 seconds, and the other through 48 feet in 4 seconds.

9. If the velocities of two bodies are as 7 : 2, and the first moves through 42 feet in a second, what space will the second body describe in the same time?

10. Compare the spaces described in a second by two bodies whose velocities are as 5 : 3.

11. If the first body describe 9 feet in a second, what space will the second body describe in a minute?

12. Compare the velocities of two bodies, of which one moves through 4 feet in a second, and the other through 40 yards in a minute.

13. Compare the velocities of two bodies, one of which moves through 6 feet in $1\frac{1}{2}$ second, and the other through 100 yards in a minute.

14. Compare the velocities of two bodies, of which one moves through 13 yards in $2\frac{1}{4}$ seconds, and the other through 260 feet in a minute.

15. The velocities of two bodies are as 5 : 2. The first describes 6 feet in a second, what space will the other describe in a minute?

16. In a lever, the velocities of P and W are as 7 : 1. What is the mechanical advantage?

17. In a system of pullies of the second kind, the number of pullies is 4. Compare the velocities of P and W.

18. In a system of pullies of the first kind, P moves through 50 inches in a second, W through 2 inches. Find the number of pullies.

19. In a wheel and axle W moves with a velocity of 4 feet in a second, P with a velocity of 160 yards in a minute. Find the mechanical advantage.

20. In a combination of levers, the mechanical advantage is represented by the ratio 60 : 7, and P describes 5 feet in a second, what is the velocity of W?

21. In a combination of a wheel and axle, and a system of 5 pullies of the second kind, P describes 8 feet in a second, W 5 feet in a minute. Find the mechanical advantage of the wheel and axle.

Prop. 19. The proof given in Wood's Mechanics is as follows:—

If not, let the line be moved parallel to itself, until it passes through the centre of gravity; then we have increased both the quantity of matter on one side of the line, and its distance from the line, and diminished both, on the other side; hence, if the body balanced itself in all positions in the former case, it cannot from the nature of the lever balance itself *in all positions* in the latter, consequently the *centre of* gravity is not in this line, which is contrary to *the supposition*.

This proof, although much shorter than that which we have given, is neither clear nor rigorous, for, in the first place, no demonstration is given of the assertion that the perpendicular distances of each particle from the line, is increased on one side of the line, and diminished on the other; and secondly, even if we allow this to be the case, the effort of each particle to turn the body round the line does not depend on the perpendicular distance of the particle itself from the line, but upon the perpendicular distance of the line in which the weight of that particle acts, from the aforesaid line.

Dr Whewell has indeed proved the proposition in this manner more fully in the Mechanical Euclid; but the proof which is there given is considerably longer than that given in the text.

We have placed this proposition after the propositions that are marked as 18 and 19 in the Schedule, because it depends upon the assumption that every body has a centre of gravity, the truth of which assumption is a simple and necessary consequence of the latter of these two propositions.

Prop. 21. We may suppose any plane figure to be composed of a number of parallel lines; and if we can prove that any other line passes through the points of bisection of each of these parallels, every one of them will balance in all positions round some point in this line, hence, the whole figure will balance round this line, and the centre of gravity of the figure will be in this line.

In this way we may determine the centre of gravity of most regular figures. For instance, a circle may be supposed to consist of a number of parallel lines, all of which will be bisected by that diameter which cuts them at right angles (III. 2), and therefore the centre of gravity of the circle will be in this diameter; similarly,

by altering the direction of the parallel lines, we might shew that the centre of gravity of the circle will be in any other diameter also, therefore it will be the point of intersection of the two diameters, that is, it will be the centre of the circle.

A cone, or cylinder may be supposed to consist of a number of circles arranged one above the other, and as the axis passes through the centre of each of these circles, it passes through the centre of gravity of each of them; hence, each of the circles will balance round the axis in all positions, and therefore the whole figure will balance round the axis in all positions, and its centre of gravity will be in the axis.

And generally the centre of gravity of the solid, formed by the revolution of any plane figure round a fixed line, or axis, will be in the axis.

Thus, a sphere is the solid formed by the revolution of a semicircle about its diameter, and its centre of gravity will therefore be in this diameter; and a hemisphere, being formed by the revolution of a quadrant round one of the radii which contain it, will have its centre of gravity in this radius.

If a body be of uniform density, and a point can be found such that the particles of matter composing the body are similarly distributed in all directions around it, that point will be the centre of gravity.

Thus, the centre of gravity of a sphere is its centre, and the centre of gravity of a ring will be the centre of the ring. In this case, as in many others, the centre of gravity is not included in the volume of the body.

22. From this proposition it follows, that the vertical line drawn through the centre of gravity of a man when he stands, must always fall within the quadrilateral formed by his feet; and the vertical line drawn through the centre of gravity of a loaded carriage, must always fall within the quadrilateral formed by the points, at which the four

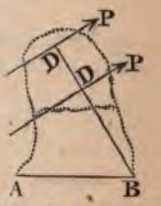
wheels are in contact with the ground; and similarly, we may determine the limits within which the centres of gravity of other bodies may move without the bodies falling over.

If a person stand sideways, close against a wall, he will find it impossible to lift the outer foot, because the wall hinders him from leaning to the other side, so as to bring his centre of gravity directly over the other foot.

By means of this proposition, we may shew that a body is rendered more unstable by increasing its height, or diminishing its base, and therefore, that those bodies are the most stable (that is, the least likely to fall over) which are of small altitudes, and rest upon large bases.

Let us first examine the effect produced by increasing the altitude of a body.

Let a force P act upon the body AB tending to turn it round the point B. Draw BD perpendicular to the direction of the force.

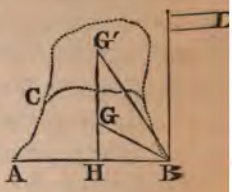


The effort which P exerts to turn the body round B, will be increased or diminished according as BD is increased or diminished; if, therefore, the force be applied at the top of the body, it will exert a greater effort to turn the body round B, when the altitude of the body is increased.

Hence, a body will be rendered more unstable if its altitude be increased, its base and weight remaining the same. Thus, if a halfpenny be laid upon a table, a force applied to the edge of the halfpenny will push it along the table, but will not upset it; if, however, a thin vertical wire be fixed to the halfpenny, the same amount of force applied at the top of the wire will be sufficient to upset it.

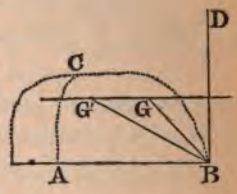
But there is another reason why a body is rendered more unstable by increasing its height. In the former case, we supposed the position of the centre of gravity of a body, ot to be *altered by* increasing its altitude, whereas in prac-

tice, when the altitude of a body is increased, *cæteris paribus*, the altitude of the centre of gravity is almost always increased also.



Let a force act on the body *ACB*, which rests on the horizontal plane *AB*, and let the force tend to turn it round the point *B*. Let *G* be the centre of gravity; draw *GH* and *BD* vertical, and join *GB*. Then, in order that the body may fall over at *B*, *GB* must be moved through the angle *GBD*. If the height of the body be increased, so that the centre of gravity be raised to *G′*, then, in order that the body may fall over at *B*, *G′B* must be turned through the angle *G′BD*.

And since $\angle G'BD$ is less than $\angle GBD$, it will be easier to turn the body over in the latter case than in the former, or, in other words, the body will be rendered more unstable by increasing its altitude, *cæteris paribus*.



Again, if the base of a body be increased, *cæteris paribus*, it is clear that the centre of gravity will be at a greater distance from the vertical line *BD*, and the angle *G′BD* will be greater than the angle *GBD*, therefore it will be more difficult to upset the body than before, or, in other words, the body will be rendered more stable by increasing its base, *cæteris paribus*.

Those bodies therefore are the most stable, whose altitude is small in proportion to their base.

Thus a waggon loaded with iron is more stable than a waggon loaded with the same weight of hay, because the centre of gravity of the waggon and its load is lower in the first case than in the second. For the same reason a column, or a long piece of timber may be carried on a waggon when laid in it horizontally, but would turn the *waggon over if* set upright in it.

The stability of a body will also, *cæteris paribus*, be affected by its weight; for it is obvious that the heavier a body is, *cæteris paribus*, the greater will be the force that is required to upset it.

A pin cannot be made to stand upon its point, because the base is so small that it is practically impossible to place the pin so that the vertical line drawn through its centre of gravity may fall within it.

But even if it were so placed, it would not stand for an instant, for its altitude is so great in proportion to the base, that the slightest force would be sufficient to upset it. The least breath of air therefore, or the slightest trembling of the surface on which it rested, though imperceptible to our senses, would be sufficient to upset the pin.

We have here considered the point of the pin to be a plane figure, but in practice it may be considered as actually a point, and therefore the pin, when it rests upon it, will be in a state of unstable equilibrium. (See the next note.)

23. It appears from this proposition that if a body be suspended from a point, it will rest whenever the centre of gravity of the body is in the vertical line which passes through the point of suspension. Now (unless the centre of gravity of a body coincides with the point of suspension) by turning the body round it may be brought either directly above or directly below the point of suspension, and there will therefore be two positions of equilibrium.

The weight of a body may be considered as a single force acting vertically at the centre of gravity, (Prop. 19), and therefore the centre of gravity will always have a tendency to fall to the lowest possible point. Hence if by disturbing the body we raise the centre of gravity, the body will return to its original position, and therefore this position of equilibrium is called *stable*.

If on the contrary by disturbing the body we make the *centre of gravity* descend, the weight of the body will

tend to move the body still farther from its original position, and therefore this position of equilibrium is called *unstable*. If the centre of gravity neither ascends nor descends when the body is disturbed, it will rest indifferently in all positions, and it is therefore said to be in a state of *neutral* equilibrium. Thus, if a body hang by a string the equilibrium is stable, if a stick be balanced on a point the equilibrium is unstable, when a sphere rests on a horizontal plane the equilibrium is neutral.

The centre of an egg will obviously be its centre of gravity, if therefore it be placed on its side, the centre of gravity will be in the lowest possible position, and the equilibrium will be stable; if it be placed on its end, the centre of gravity will be in the highest possible position, and therefore, if the body be disturbed ever so little, the centre of gravity will descend, and the equilibrium will therefore be unstable.

When a rigid body is suspended from a fixed point, the centre of gravity will, if we move the body, describe a circle round this fixed point, and therefore when the centre of gravity is directly below the point of suspension, it is always in the lowest possible position, and the equilibrium will always be stable; when the centre of gravity is directly above the point of suspension, it will always be in the highest possible position, and the equilibrium will always be unstable. But if a body rests upon a plane, the point of support changes as we move the body, and therefore the position may be one of stable equilibrium, although the centre of gravity be above the point of support.

We now proceed to shew why a pair of scales, equally loaded, will rest only when the beam is horizontal.

The centre of gravity of the two weights always lies in the vertical line which passes through the centre of the beam, (*vid.* example 5). If therefore, the centre of the beam (which will obviously be its centre of gravity) were also the point of suspension, the pair of scales would rest *indifferently in* any position, but in practice the point of

suspension is always above the centre of the beam, and therefore the balance will rest, only when the vertical line drawn through the point of suspension passes through the centre of the beam, which, if the balance be accurately constructed, can only be the case when the beam is horizontal.

If the centre of the beam be above the point of suspension, the equilibrium will be unstable; if it be below, the equilibrium will be stable; and if the system be made to oscillate, it will return to its original position.

This proposition furnishes us with an easy practical method of determining the centre of gravity of a body.

For if the body be suspended from any point, and when it rests, a vertical line be drawn through the point of suspension, the centre of gravity will be in this line.

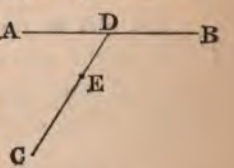
If the body be again suspended from any other point, and another vertical line be drawn through this point, the centre of gravity will be in this line also.

Hence the centre of gravity will be the point of intersection of these two lines.

EXAMPLES.

1. Find the centre of gravity of 3 equal points.

Let ABC be the points, w the weight of each.



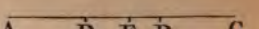
Join AB, and bisect it in D, therefore D will be the centre of gravity of A and B, which will produce the same effect as $2w$ at D.

Join DC, and take $DE : DC :: w : 3w$;

$\therefore DE : EC :: w : 2w$, and E will be the centre of gravity of C and $2w$ at D,

$\therefore$ it will be the centre of gravity of C, A, and B.

2. If the three points are in the same right line.



Then if $AB = a$, $BC = b$.

$$AD = \frac{a}{2}, \quad AE = AD + DE.$$

$$\therefore AE = \frac{a}{2} + \frac{DC}{3} = \frac{a}{2} + \frac{1}{3}(DB + BC)$$

$$= \frac{a}{2} + \frac{1}{3}\left(\frac{a}{2} + b\right) = \frac{a}{2} + \frac{a + 2b}{6} = \frac{4a + 2b}{6};$$

$$= \frac{2a + b}{3}.$$

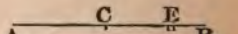
3. If $BC = AB$, i. e. $b = a$.

$$AE = \frac{3a}{3} = a = AB;$$

$\therefore$ E coincides with B, and B will be the centre of gravity.

4. The weight of a uniform rod is 3 lbs., if an additional weight of 5 lbs. be suspended at one end, where will be the centre of gravity of the whole system?

Let AB be the rod, bisect it in C. C will be the centre of gravity of the rod; and the weight of the rod will produce the same effect as 3 lbs. suspended at C;



divide CB in E so that $CE : CB :: 5 : 8$,

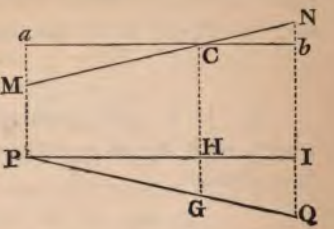
$\therefore$ dividendo $CE : EB :: 5 : 3$.

Hence, E will be the centre of gravity of 3 lbs. suspended at C, and 5 lbs. suspended at B, that is, it will be the centre of gravity of the system.

5. At the ends of a straight lever MN are suspended two weights P and Q. Find the position of the centre of gravity when the weights are in equilibrio.

Join PQ; draw CG vertical; G, their point of intersection shall be the centre of gravity.

Draw aCb, PHI horizontal.



$P : Q :: Cb : Ca$, since there is equilibrium. (Prop. 6.)

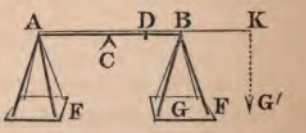
$:: HI : PH$. (Euclid, I. 34.)

$:: QG : PG$ by similar triangles;

$\therefore$ G is the centre of gravity.

6. If a man sitting in one scale of a weighing machine, press with a stick against any point of the beam between the point from which the scale is suspended, and the fulcrum, he will appear to weigh more than before.

If a man sitting in the scale F press with a stick against any point D between C and B, he will push the scale F in which he sits, farther from C; the centre of gravity, therefore, of himself and the scale F will be moved from G to some point G' further from C; and therefore, if $G'K$ be drawn perpendicular to CB, the arm CK at which his weight acts, will be greater than CB, the arm at which it acted before. The scale F, therefore, will preponderate.

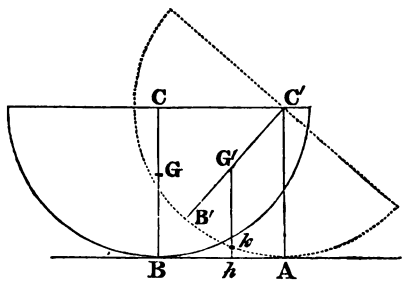


If he press against B, he will not alter the position of

his centre of gravity. But if he press against any point beyond B, (as for instance K) he will push himself nearer to the fulcrum C, and his weight will act at a shorter arm than before, therefore the scale E will preponderate, and he will appear to weigh less than before.

7. If a solid hemisphere be placed on a horizontal plane, with its axis vertical, the position is one of stable equilibrium.

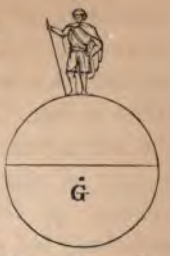
Let the annexed figure represent a section of the hemisphere, and let the point G in the axis CB, be the centre of gravity; then, if the hemisphere be moved round into the position represented by the dotted lines, the line AB will touch the circle, and therefore (III. 17) $C'A$ will be vertical.



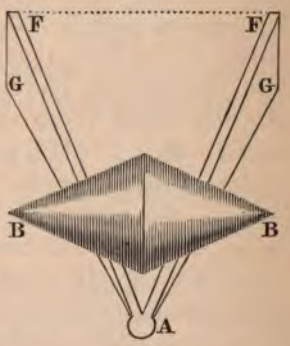
If $G'kh$ be drawn vertical, cutting the circumference in k, and AB in h, $G'h > G'k$.

But $G'k > G'B'$ (3, 7); $\therefore Gh > G'B'$ or GB.

If therefore we move the hemisphere, we make the centre of gravity ascend, and the body will consequently return into its original position, which will therefore be one of stable equilibrium.



If a light figure be placed upon a hemisphere so that the centre of gravity of the whole is below the centre of the hemisphere, the position will still be one of stable equilibrium; and such a body may be made to oscillate in any direction upon a horizontal plane without falling over. The same thing will occur if the body be placed on the top of a sphere, provided the lower half of the sphere be so much heavier than the upper, that the centre of gravity of the whole falls below the centre of the sphere.



8. If a double cone or spindle *BB* be placed upon two inclined planes *FA*, *FA*, whose height *FG* is small, it will of itself move from *A* towards *FF*.

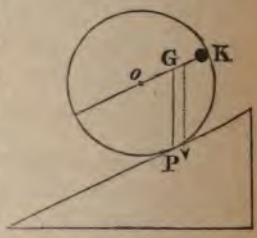
The centre of the axis of the spindle will clearly be its centre of gravity, therefore, if *FF* be just equal to the axis of the spindle, when the spindle comes to *FF*, its centre of gravity will be in *FF*, and the height of the centre of gravity will be *FG*.

When the spindle is at *A*, the height of its centre of gravity above the horizontal plane in which the lines *AG*, *AG* lie, is the radius of the circle which forms the common base of the two cones. If therefore *FG* be less than this radius, the centre of gravity will descend from its highest position, viz. that at *A*, to its lowest position, viz. that at *FF*, and the spindle will roll up the planes.

9. To shew how a cylinder may be made to roll up an inclined plane.

If a cylinder *A* be made of light wood, with a small cylinder of lead *K*, going through it close to the surface, the

centre of gravity of the body will no longer be the centre of its axis, but some point G in the radius Ko, between K and the centre o. If such a cylinder be placed on an inclined plane so that the vertical line through G* is above the point P at which the cylinder rests upon the plane; the weight of the cylinder acting in this line will cause the cylinder to roll up the plane, provided the friction be sufficient to prevent sliding. And the cylinder will continue to roll up the plane, until G has come into its lowest possible position, which will be in the vertical line passing through the point of support. Although the cylinder seems to ascend in this case, the centre of gravity G really descends.



10. Fig. 1. If a piece of wood (AB) be placed upon a table so that its centre of gravity G is beyond the edge (M) of the table; its weight acting at G will turn it round M, and it will therefore fall; but if a heavy body C, be joined to AB so as to bring the centre of gravity of the whole K, to any point beneath the table; the weight of the two bodies acting at K will be supported by the table; and they will therefore remain at rest.

Fig. 2. Thus, if a bucket be suspended from the piece of wood, and we fix another stick with one end in a notch in the piece of wood, and the other against the inside of the pail at the bottom, the centre of gravity of the whole may be brought under the table, and will in that case be supported by it.

* G, in the figure, ought to be placed at the point where the dotted line *meets the* diameter.

Fig. 3. Again, if two knives be stuck into the piece of wood, the centre of gravity of the whole will be brought nearer to the end *A*, and it will therefore be supported by the table, and may even be balanced on the rim of a glass, or a sharp point.

11. Find the centre of gravity of a parallelogram.

12. At two of the angular points of an equilateral angle are suspended weights of one pound, at the third angular point is suspended a weight of two pounds. Find the centre of gravity of the system.

13. Why does a porter with a load on his back lean forwards?

14. Why does a nurse with a child in her arms lean backwards?

15. In ascending a hill we appear to lean forwards, in descending to lean backwards. Do we do so in reality?

16. A weight is suspended to a pole, and carried on the shoulders of two men of unequal heights, the point of suspension being equally distant from each. Does each man sustain an equal portion of the weight?

17. Why does a man continually shift his hand when he balances a stick upon it?

18. Explain the phenomenon of a billiard ball moving up two inclined rods.

19. What is the use of a rope dancer's pole?

20. If a weight be suspended by a string from the middle point of a pole, and the pole be carried by two men, so that the ends of the pole rest upon their shoulders; shew that they bear an equal portion of the weight, whether they ascend or descend a hill.

21. Is this the case, when the weight is attached to the pole by a rigid rod, instead of a flexible string, the rod being perpendicular to the pole?

22. Why is a solid cylinder resting on a horizontal plane more difficult to upset, than a hollow cylinder of the same dimensions?

23. Why are lofty sails more likely to upset a vessel than low ones?

Do they propel a vessel better than low ones?

24. Why does a man, when he rises from a chair, stoop forwards?

25. Explain why it is more dangerous to place luggage on the top of a coach, than in the body.

26. A heavy uniform rod, with a weight of 12lbs. at one end, balances in a horizontal position round a fulcrum 9 inches from that end. If the length of the rod be 4 feet, what is its weight?

27. Why is it necessary that a table should have three feet at least?

28. How is it that an inclined tower, such as that at Pisa, does not fall, although its top hangs 12 feet over its base?

29. Supposing such a tower to be built in the form of an oblique cylinder, so that the slant side is to the height as 5 : 4; what is the greatest height to which it may be built in theory, if the radius of its base be 30 feet?

HYDROSTATICS.

1. SAND and many other powders, may be divided in any direction, and their parts can be moved among one another by very small forces; but not by *any* force however small. They do not therefore come under the definition of a fluid, although they possess the properties of fluids to a certain extent.

When a pressure is applied to a solid at any point, the solid may be kept at rest by applying an equal and opposite pressure to the opposite point of the solid. If a pressure be applied to a fluid contained in any close vessel, an equal pressure must be applied to *every* point of the surface of the fluid, in order to preserve equilibrium. (Prop. 1.)

If the vessel were filled with sand, we should find that the sand would transmit pressure in different directions, but not equally in all directions.

Again, sand will flow from a higher to a lower level, but it will not, like a fluid, flow to a perfect level. (Prop. 2.)

If the reader will imagine a vessel to be filled with spherical shot, whose surfaces are free both from friction and cohesion, he will have a pretty correct notion of the particles of a fluid, which would differ from these shot only in being exceedingly small.

2. Fluids are said to be compressible when their dimensions are diminished by increasing the pressure upon them; they are said to be elastic, when they recover their former dimensions, if the pressure to which they have been subjected is removed.

As all fluids that are compressible are also elastic, the terms compressible and elastic, incompressible and inelastic may in hydrostatics be considered as synonymous.

3. If a tube be filled to a certain height with water or any other fluid, and placed under the receiver of an air-pump; when the air in the receiver is exhausted the fluid will rise in the tube. Again, if the tube be placed under the receiver of an air-pump and subjected to an increased pressure, the fluid will sink in the tube.

If a bottle filled with water and corked tight, be plunged to a great depth in the sea, on drawing up the bottle, the water within it will be found to have acquired a brackish taste, whence we may conclude that in such cases the cork is forced in by the pressure so that the sea water enters and mixes with the water in the bottle.

These experiments satisfactorily prove that water and other fluids are in some degree compressible.

Water, however, and the rest of the fluids which are commonly called liquids, are susceptible of compression in so slight a degree, that for all practical purposes, they may be considered as absolutely inelastic.

Prop. 1. Upon the principle proved in this proposition, depend the contrivances called safety valves.

Suppose a steam boiler or a condenser to be so constructed that it cannot bear at every part of its surface a greater pressure than 25 lbs. to the square inch.

If a valve whose area is a square inch be loaded so that it requires a pressure of 25 lbs. to raise it, and placed at *any part whatever* of the boiler; then, if any portion of the boiler sustain a pressure of more than 25 lbs. to the square inch, the valve will sustain the same pressure; it will therefore open, and the steam will escape until the

pressure on the sides of the boiler is reduced within the proper limits; when the valve will again close by its weight.

PROP. 3. Upon this principle are constructed the instruments that are used in levelling.

The common level consists of a tube with its two ends turned up and open. It is filled with water, or mercury, and in each end is placed a float carrying an upright sight. The sights are at equal distances above the floats, therefore the line which joins them will always be parallel to the surface of the fluid.

The surface of the fluid will always be horizontal, therefore this line will always be horizontal, however the instrument may be held; and any object that is seen in this line will be on the same level with the instrument.

PROP. 2, 3, 4. In these propositions the pressure of the fluid is calculated as if there were a vacuum above its surface. In order, therefore, to find the actual pressure at any point, we must add the atmospheric pressure, to the pressure which is due to the fluid itself. The atmospheric pressure is very nearly equal to a pressure of 15 lbs. on every square inch.

If the area of the bottom of a vessel be a square foot, the pressure on the bottom will be less in a vacuum, by 144×15, or 2160 lbs. than it is in air. This enormous pressure is not perceptible, under ordinary circumstances, because the air exerts an equal pressure upwards on the outside of the vessel.

PROP. 2, 3. If we suppose the surface of a fluid at rest to consist of a number of different plane surfaces, the depth of any point below the surface will be different according as it is measured from one or another of these planes. The same thing will occur if we suppose the surface of a

fluid to be curved, for a curved surface may be conceived to be made up of a number of plane surfaces of indefinitely small extent. Now the enunciation of Prop. 2 evidently assumes that the depth of any point below the surface has only one value, which cannot be the case unless the surface of the fluid be a plane surface.

The enunciation therefore assumes that the surface of a fluid at rest is a plane surface: and hence, this proposition ought, in strictness, to be placed not before Prop. 3, but after it, as is done in almost every work upon hydrostatics.

In fact, Props. 2 and 3 of the Schedule comprise three distinct theorems, which in Miller's Hydrostatics are arranged in the following order:

(1) In a fluid at rest, the pressures at all points in the same horizontal plane, are equal.

(2) The surface of a fluid at rest is horizontal.

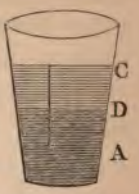
(3) In a fluid at rest, the pressure on any particle is proportional to its depth below the surface of the fluid.

The reader will see that both of the first two theorems are comprised in the proof of Prop. 3 of the Schedule.

In these propositions the fluids employed are always supposed to be of uniform density; if, however, the fluid in a vessel be not of uniform density (as is the case when various liquids of different specific gravities, which do not mix together, are poured into the same vessel) the propositions will still be true, with the exception of Prop. 2.

Prop. 1 will be true whether the density be uniform or not, because the experiment on which it depends has no reference to the weight of the particles which compose the fluid mass.

PROP. 2 will require some slight modification.



The pressure at any point will depend upon its depth below the surface, but it will not be proportional to the depth. Thus, if the vertical line AC meet the surface of the denser fluid in D, and that of the lighter fluid in C, the pressure on A will be equal to the weight of the fluid particles in CD + the weight of the particles in AD, but it will not be proportional to the whole depth AC, because the particles in CD are not so heavy as those in AD.

PROP. 3 will be true whether the density be uniform or not.

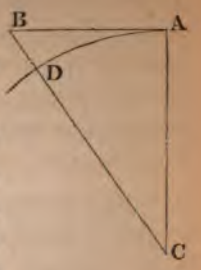
For in the proof of the proposition, P and Q may be at any depth whatever below the surface; let them be taken anywhere within the upper fluid, and the rest of the proof will apply whether the density be uniform or not.

PROP. 4 is evidently true whether the density be uniform or not, as also will be

PROP. 6; but the reader must recollect that the weight of the fluid displaced will not be the same as it would be if its density were uniform.

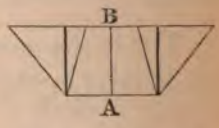
PROP. 2. In Prop. 3, the lines in which the force of gravity acts are supposed to be parallel to each other, and hence it has been proved that the surface of a fluid at rest is a horizontal plane. In reality, however, these lines are not parallel, but intersect at the earth's centre, and therefore the surface of a fluid at rest is a spherical surface and not a horizontal plane. But if the extent of the surface of the fluid be small; (the angle contained by these lines will be so small that) the lines may be considered as parallel, and the surface of the fluid will not therefore differ materially from a horizontal plane.

The object in levelling is to determine a number of different points, whose distances from the centre of the earth C, are equal.



If, therefore, B is a point in the same horizontal line as A, when the distance AB is considerable, CB cannot be considered as equal to CA. A correction must therefore be made for BD, which is technically termed the depression.

Prop. 4. If the sides of the vessel are not vertical, the pressure on any point A of the bottom of the vessel will be the weight of the column of fluid particles AB; and the pressure will be the same at all points of the bottom of the vessel, therefore the whole pressure on the bottom, will be equal to the weight of a column of fluid whose base is the bottom, and whose altitude is the depth of the bottom below the surface of the fluid.



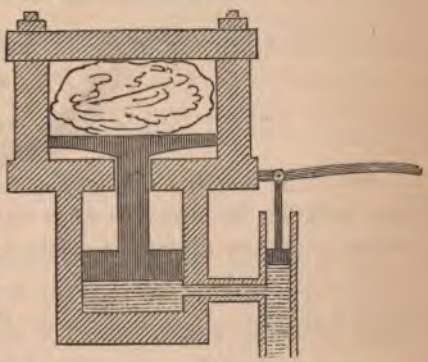
The pressure on the bottom, therefore, is not altered by the form of the upper part of the vessel so long as the altitude of the fluid remains the same; and if the area of the surface of the fluid be less than that of the bottom, the pressure on the bottom will be greater than the weight of the fluid.

This fact has sometimes been called the hydrostatic paradox; as well as the fact explained in Prop. 5.

Prop. 5. If in the hydrostatic paradox the weight of the small column be exchanged for the pressure of a small piston, and the place of the moveable board be supplied by a large moveable piston; the pressures exerted by the pistons will be proportional to their areas. If an additional pressure be applied to the smaller piston, the large

piston will ascend, and press against any obstacle which is placed above it. This is the principle upon which is constructed the instrument called the hydrostatic press. The smaller piston is worked by means of a lever, and the larger presses any object placed upon it, against an iron framework in which the instrument is fixed.

The instrument is used in packing, where it is necessary to compress bulky articles into a small compass; for trying the strength of cables, and other purposes, where it is necessary to exert a great pressure through a small space.



There is no mechanical contrivance so simple or so powerful as this; the space it occupies is very small, and the friction is inconsiderable. If the area of the smaller piston be a square inch, and that of the larger a square foot, a force of 1lb. will produce a pressure of 144lbs. The principle of virtual velocities applies to this as well as to all other machines, for the ascent of the large piston is as much slower than the descent of the smaller one, as its area exceeds the area of the smaller.

A full account of the different contrivances used in the construction of this beautiful machine may be found in Webster's Hydrostatics.

For the machine in its present state of perfection, we are indebted to Mr Bramah, whose name it bears, although the merit of its original invention has been attributed to Simon Stevin of Bruges.

N. B. In the following examples, the pressure of the atmosphere upon the surface of the fluid is not taken into

account, the pressures in the interior of the fluids being calculated as if there were a vacuum above the surface.

EXAMPLES.

1. Compare the pressures of a fluid at the depths of 2 inches and 18 feet, below the surface.

Here the pressures are as 2 inches : 18 feet,

2 inches : 216 inches,

or, as 1 : 108.

2. The weight of a cubic foot of water is 1000 oz., what will be the pressure on a square inch at a depth of 18 feet below the surface of a lake?

The required pressure = pressure of a column of water, whose base is a square inch, and height 18 feet,

= weight of 1×216 cubic inches of water,

$= 216 \cdot \frac{1000}{1728}$ oz., since 1728 cubic inches = 1 cubic foot,

$= \frac{1000}{8}$ oz.,

= 125 oz.

3. Mercury is poured into a vessel, the bottom of which is horizontal; the area of the base is 2 square feet and the depth of the mercury is 6 inches. Find the pressure on the bottom, the density of mercury being 13.5.

A cubic foot of water weighs 1000 oz., the density of water is 1, that of mercury 13.5; therefore a cubic foot of mercury weighs 13500 oz., and pressure on the bottom

= weight of $2 \times \frac{1}{2}$ cubic feet of mercury,

= weight of a cubic foot of mercury,

= 13500 oz.

4. A small tube of uniform bore is inserted in the bung of a hogshead full of water; the area of a horizontal section of the tube is a square inch, and that of the top of the hogshead is 2 square yards. The tube is filled with water to the height of 36 feet; find the weight of the water in the tube, and the pressure of the fluid upwards, on the top of the hogshead.

The weight of the fluid in the tube = weight of 1×432 cubic inches of water,

= weight of 432 cubic inches of water,

$$= 432 \times \frac{1000}{1728} = 250 \text{ oz.},$$

and this will be the pressure on a section of the tube, i.e. on a square inch.

Therefore pressure on a section of the tube : pressure on the top of the hogshead

:: 1 inch : 2 square yards,

:: $1 : 2 \times 9 \times 144$;

$\therefore$ pressure on the top of the hogshead = $2 \times 9 \times 144 \times$ pressure on a section of the tube

$= 2 \times 9 \times 144 \times 250$ oz.,

= 40500 lbs.

Thus by means of about 16 lbs. of water we might produce a pressure of nearly 20 tons. Such a pressure would be sufficient to burst the strongest hogshead.

If a heavier fluid were used, a smaller column would produce the same effect. Thus, if mercury were used, a column of about one thirteenth the height would produce the same amount of pressure on the top of the hogshead.

If a large cavity in the bowels of a mountain were to be filled with water by a small crack of considerable height,

and the crack itself should get full of water, it would be impossible to fix any limits to the convulsion which might ensue.

The instrument described in Prop. 5, is sometimes called the hydrostatic bellows; and the weight upon the board may be supported and raised by blowing into the tube, instead of pouring water into it.

5. If two fluids of different densities, which do not mix, be poured into a bent tube, will they stand at the same level in both legs?

In this case the pressures of the two columns of fluid in the two legs must counterbalance each other, and therefore be equal.

A longer column therefore of the lighter fluid will be required to counterbalance the fluid in the other leg, than would be required if the tube were entirely filled with the denser fluid. Therefore the lighter fluid will stand at a higher level than the denser.

6. Explain the reason why an egg is not crushed by the pressure of water, however deep it may be plunged beneath the surface.

Fluids press equally in all directions, and therefore, however great the pressure of the fluid may be, every part of the egg is equally pressed, and it will not be crushed or altered in shape.

7. Explain the reason why a mass of water falling upon a man from a considerable height, does him no injury. Why is not this the case with a mass of ice?

According to the definition, the particles of a fluid may be divided by any force however small, and the resistance of the air produces this effect, so that the water falls in a kind of shower, and does the man no injury.

But a mass of ice, being solid, would not be separated by the resistance of the air, and would in all probability fracture his skull.

8. A body specifically heavier than a fluid may be made to swim in that fluid, and a body specifically lighter may be made to lie at the bottom.

If a plate of metal *m*, be pressed against the mouth of a glass tube *gg*, and the whole immersed in a vessel of water *ww*; then if there is no water in *gg*, the downwards pressure on *m* will be its weight added to the pressure of the air in *gg*.

Now, by sinking *gg* to a sufficient depth, the pressure of the water upwards on *m* may be made greater than the pressure downwards, and it will therefore retain *m* in its place; although its specific gravity is greater than that of the fluid.

Again, if a plate *ll*, of any substance lighter than water be placed at the bottom of the vessel, and a quantity of water be poured gently into the vessel, provided the surfaces in contact are made so smooth, that the water cannot insinuate itself between them; the water will exert no pressure on the under surface of the plate *ll*, which will remain at the bottom, although its specific gravity be less than that of the fluid.

This experiment may be tried with a piece of cork, by fixing to its under side a thin and smooth plate of brass.

9. Compare the pressures in the interior of a fluid at rest, at the depths of 3 inches and 4 feet.

10. A cubic inch of mercury weighs 8 oz., what will be the pressure on the square inch in a vessel of mercury at a depth of two feet below the surface?

11. What will be the pressure on the bottom of a vessel filled with mercury; the height of the mercury being

6 inches, and the area of the bottom (which is horizontal) being a square foot.

12. The atmospheric pressure is 15 lbs. on the square inch; compare the pressures in a vessel of mercury at the depths of 10 and 50 inches below the surface.

13. In the hydrostatic bellows the area of the tube is $\frac{1}{4}$ of a square inch, and the height of the column of water contained in it above the surface of the board is 30 feet. If the area of the board is a square foot, what will be the weight of the water in the tube, and what weight must be placed upon the board in order to counterbalance it?

14. The shape of a soft substance, such as a lump of wax, is not altered by the pressure of a fluid however deep it may be immersed in it. Explain the reason why this is the case.

15. A plate of metal is pressed against the mouth of a glass tube, which it exactly fits, the thickness of the plate being one inch; and the specific gravity of the metal being 12. How deep must the tube be plunged in a vessel of water, in order that the plate may be retained in its place by the pressure of the water alone, the experiment being supposed to take place in vacuo?

16. What will be the pressure on the bottom of a vessel filled with water, (1) in vacuo; (2) in air; the area of the bottom (which is horizontal) being 2 square feet, and the depth of the water being one foot?

Will a vessel which is strong enough to bear the pressure in the first case, necessarily be strong enough to bear the pressure in the second case?

17. Why do divers experience no inconvenience from the pressure of the water, even when they descend to considerable depths?

18. A spring-head, and the top of a distant tower, are ascertained to be in the same horizontal line. Can water

be conveyed from the spring-head to the top of the tower, by means of a series of connected pipes?

19. Compare the pressure at a depth of 17 feet in a vessel of water with the pressure at a depth of 15 inches in a vessel of mercury: if the density of water is to the density of mercury as 6 : 68.

PROPS. 6, 7, 8, 9. If a vessel full of water be weighed in a balance, and any body whatever be held in the vessel so as not to touch the sides or bottom of the vessel, although the greater part of the water may have been made to run over the sides, the scales will still continue balanced, and this will be the case, whatever may be the specific gravity of the body which is thus held in the water. Thus a block of wood will keep the scales balanced just as well as a block of lead of the same size.

For by proposition A, The pressure of the body downwards = the pressure of the fluid upwards on the body = the weight of the fluid displaced = the weight of a quantity of fluid of the same bulk as the body.

On the same principle, if the vessel be not full of water, so that, when the body is immersed the water rises in the vessel instead of running over the sides; the pressure of the body downwards will still be equal to the weight of a quantity of fluid of the same bulk as the body; and the weight of the water will be increased by this additional pressure.

Thus, if a glass of water rest upon a table, and a cubic inch of lead be suspended in the water by a string, the table will sustain an additional pressure, which will be equal to the weight of a cubic inch of water.

If a cubic inch of any substance whose specific gravity is less than that of water, were forcibly held in the water, it would add to the weight of the water as much as the lead does.

When a body is wholly immersed in a fluid, the fluid above the body presses it downwards, and the fluid below the body presses it upwards; but as the pressure of the fluid at any point is proportional to the depth of that point below the surface, the fluid beneath the body, being lower down than that above the body, will exert the greater pressure. The effective pressure of the fluid will, therefore, be equal to the difference of these pressures, and will always act vertically upwards. This effective pressure (which we have shewn to be equal to the weight of a quantity of fluid whose bulk is equal to that of the body) will press the body upwards; and the body will press downwards with its own weight. If therefore the weight of the body be greater than the weight of an equal bulk of fluid, the body will descend; if they be equal, the body will remain at rest wherever it be placed in the fluid; if the weight of the body be the less, the body will rise.

Props. 10, 11. If we can ascertain accurately the magnitude of a body, we can find its specific gravity by weighing it. For the weight of an equal bulk of water may be calculated, and the specific gravities of the body and of water will be as the weights of these equal bulks. But if the shape of the body be irregular, it is impossible to ascertain its magnitude by measurement, and we must therefore have recourse to some other method. Now when a body is immersed in a vessel full of any kind of fluid, it displaces a quantity of fluid whose bulk is equal to its own, and if we were to measure the fluid which runs over the sides of the vessel, we should be able to ascertain accurately the magnitude of the body immersed, and might calculate the weight of an equal bulk of water.

But Prop. A furnishes us with a still simpler method, for the weight lost by the body when weighed in water gives us the weight of an equal bulk of water at once.

On this principle is constructed the hydrostatic balance,

by means of which we are able to determine the specific gravities of solids. We might ascertain the specific gravities of fluids by means of the same instrument; for if the same body were weighed in two fluids, the weight lost in each would be equal to the weight of an equal bulk of that fluid; and the specific gravities of the fluids would be proportional to these weights.

The hydrometer, however, is more generally used for the latter purpose, inasmuch as it is a simpler and more convenient instrument.

The principle proved in Prop. A was discovered by Archimedes, who employed it to solve the well-known problem of the adulterated crown.

Hiero, king of Syracuse, had given a workman a certain quantity of gold to make into a crown. As it is always necessary to alloy gold with some other metal before it can be wrought, the workman added a certain quantity of silver. When the crown was brought back, the king, suspecting that the workman had alloyed the gold with more silver than he owned, and had thereby cheated him of so much gold, (for it is probable that the actual weight of the crown was the same as it would have been, if the gold had been fairly alloyed) requested Archimedes to find out how much gold and how much silver were in the crown. The way in which he solved the problem will be seen in the examples.

EXAMPLES.

1. A cylinder of cork floats in a vessel of water, the specific gravity of cork is .25, that of water is 1. What part of the cylinder will be above the surface?

Let m be the number of cubic inches above the surface

n..... below .....

$(m + n) \times .25$ = the weight of the cylinder.

$n \times 1$ = the weight of the fluid displaced,

and these are equal by proposition 6;

$$\therefore\ n = (m + n) \times .25 = (m + n) \times \tfrac{1}{4}.$$

$= \frac{1}{4}$ of the content of the cylinder.

Therefore one quarter of the cylinder will be below the surface, and three quarters of the cylinder will be above the surface.

2. A cylinder floats in a fluid whose specific gravity is 13.5, with two thirds of its bulk above the surface. What is the specific gravity of the cylinder?

Let S be the specific gravity of the cylinder,

Then, since $\frac{1}{3}$ of the cylinder is below the surface,

by Prop. 6, $S \times 1 = 13.5 \times \frac{1}{3}$,

$S = 4.5$, the specific gravity of the cylinder.

3. A cylinder, whose specific gravity is .6, floats in a fluid with $\frac{3}{5}$ of its bulk below the surface. Find the specific gravity of the fluid.

If S = the specific gravity of the fluid,

$$S \times \frac{3}{5} = \text{weight of the fluid displaced,}$$

$$= \text{weight of the cylinder, (Prop. 6),}$$

$$= 1 \times .6,$$

$$\frac{3S}{5} = .6, \qquad \frac{S}{5} = .2,$$

and $S = 1$, the specific gravity of the fluid.

4. A cylinder floats in a fluid with $\frac{2}{7}$ of its bulk above the surface. Compare the specific gravities of the cylinder and the fluid, (by Prop. 8.)

The specific gravity of the body : the specific gravity of the fluid

:: the part immersed : the whole body

$$:: 1 - \frac{2}{7} : 1,$$

$$:: \frac{5}{7} : 1,$$

$$:: 5 : 7.$$

5. A lump of lead which weighs 22 lbs. loses 2 lbs. when weighed in water. Compare the specific gravities of lead and water.

By Prop. 9, the specific gravity of water : the specific gravity of lead,

$$:: 2 : 22,$$

$$:: 1 : 11.$$

6. A lump of gold weighs 152 lbs. in air, and 44 in mercury. Find the specific gravity of mercury, that of gold being 19.

Specific gravity of mercury : specific gravity of gold :: weight lost : whole weight,

specific gravity of mercury : 19 :: 108 : 152,

$$152 \times \text{specific gravity of mercury} = 19 \times 108;$$

$$\therefore \text{specific gravity of mercury} = \frac{19 \times 108}{152} = \frac{108}{8},$$

$$= 13.5.$$

7. A lump of marble weighs 27 lbs. in air, and 17 in water. Find its specific gravity.

By Prop. 9.

Specific gravity of water : specific gravity of marble :: weight lost : whole weight,

$$:: 10 : 27;$$

$\therefore$ 27 × specific gravity of water = 10 × specific gravity of marble,

$$\text{specific gravity of marble} = \frac{27}{10} \times \text{specific gravity of water}$$

$$= \frac{27}{10} \times 1 = \frac{27}{10} = 2.7.$$

8. A block of wood weighs 3 lbs. in air; as it will not sink in water, a piece of lead is added to it as a sinker; the lead weighs 6 lbs. in water, and the two together weigh 5 lbs. in water. Find the specific gravity of the wood.

By the formula of Prop. 10.

$$S = \frac{w}{w + w_2 - w'} \times S'.$$

Here S' = specific gravity of water = 1,

w = weight of wood in air = 3 lbs.,

w_2 = weight of lead in water = 6 lbs.,

w' = weight of the two together in water = 5 lbs.;

$\therefore$ S = the specific gravity of the wood $= \frac{3}{3 + 6 - 5} \times 1$

$$= \frac{3}{4} = .75.$$

9. A lump of iron weighs 10 lbs. in air; a lump of platina which weighs 15 lbs. in air, and 9 in mercury, being used as a sinker, the iron and platina together are found to weigh 1 lb. in mercury. Find the specific gravity of iron, that of mercury being 13.5.

This might be found from the formula, as in the last example; but we proceed to find it independently.

If S = the specific gravity of the iron, m its bulk,

weight of fluid equal in bulk to the two = weight lost by the two = $10 + 15 - 1 = 24$,

..... platina = weight lost by the platina = $15 - 9 = 6$,

$\therefore$ weight of fluid equal in bulk to the iron alone = 18,

$$\therefore m \times 13.5 = 18,$$

but $m \times S = 10$;

$$\therefore \frac{m \times S}{m \times 13.5} = \frac{10}{18},$$

$$S = \frac{10}{18} \times 13.5 = \frac{135}{18},$$

$= 7.5$ = the specific gravity of iron.

10. A hydrometer floats in two fluids, with 50 and 60 divisions of the stem above the surface. If each division contains $\frac{1}{1000}$ of the whole bulk of the instrument, compare the specific gravities of the fluids.

If S, S' be their specific gravities

$(1000 - 50)S$ = weight of fluid displaced in the first case,

= weight of hydrometer, (Prop. 6.)

= weight of fluid displaced in the second case, (Prop. 6.)

$= (1000 - 60)S'$;

$950S = 940S'$.

$\therefore S : S' :: 940 : 950 :: 94 : 95.$

11. The same hydrometer floats in alcohol with 11 divisions above the surface, in ether with 80. Find the specific gravity of ether, that of alcohol being .8.

$(1000 - 11) \times .8$ = weight of alcohol displaced = weight of hydrometer,

= weight of ether displaced,

$= (1000 - 80) \times S$, if S = the specific gravity of ether,

$989 \times .8 = 920S$,

$$S = \frac{791.2}{920} = .86,$$ the specific gravity of ether.

12. When a body is immersed in a fluid, what is the effective pressure of the body?

The fluid presses the body upwards with a force equal to the weight of the fluid displaced, and the body presses downwards with its weight.

The difference, therefore, of these two forces is the effective pressure of the body, and it will act in the direction of the greater force.

13. Two bodies of different densities are in equilibrio when suspended by strings from the scales of a balance, and immersed in a fluid. Will they balance when immersed in a fluid of different density?

Each of the bodies loses by being immersed a part of its weight equal to the weight of the fluid which it displaces. If, therefore, the bodies are immersed in a fluid of different density, the weight lost by the bodies will not be the same as before, and the bodies will no longer balance.

If removed to a denser fluid, the weight lost by the larger body will exceed that lost by the smaller, still more than before, and the smaller body will preponderate. If removed to a rarer fluid, the weight lost by the larger will not exceed that lost by the smaller so much as before, and the larger body will preponderate.

Thus, if two bodies balance when weighed in air, they will not necessarily balance when weighed in vacuo; but the weight lost in air is so small, that the weight of a body in air may be considered as the true weight.

14. The volume of a certain weight of gold is m, that of the same weight of silver is m', and that of the same weight of mixed metal is μ. Find the volume of the gold contained in the compound.

Let x = the volume of the gold contained in the compound.

$\therefore \mu - x =$ ..... silver .....

and let S, S' be the specific gravities of gold and silver.

$xS + (\mu - x)S' =$ weight of the compound $= mS = m'S'$;

$$\therefore (\mu - x)S' = (m - x)S, \text{ and } S' = \frac{mS}{m'};$$

$$\therefore \text{ by substitution } (\mu - x)\frac{mS}{m'} = (m - x)S;$$

$$(\mu - x)m = (m - x)m',$$

$$\text{hence } x = \frac{m' - \mu}{m' - m} \cdot m.$$

It was upon this principle that Archimedes solved the problem of the crown. He procured a lump of gold, and another of silver, of the same weight as the crown, and ascertained the quantity of gold in the crown, by comparing the bulks of the three bodies.

It is probable however that he did not solve the problem generally by means of an algebraical formula as we have done in the example; but confined himself to the particular example which was before him, and solved the problem approximately for that single case.

Thus, since silver is lighter than gold, the bulk of the crown would be greater, or less, according as more or less silver entered into the composition. Let us suppose that the volume of the lump of gold was 10 cubic inches, and that of the lump of silver 18 cubic inches, then if the mixture was one half silver, and one half gold, its bulk would have been $5 + 9$ or 14 cubic inches. If it was two thirds silver and one third gold, its bulk would be, $\frac{10}{3} + \frac{2}{3} \cdot 18$, or $15\frac{1}{3}$ cubic inches, and so on. By measuring the bulk of the crown, therefore, he could ascertain with tolerable accuracy the quantity of gold contained in it.

Archimedes' chief difficulty in the solution of the problem was how to find the exact bulk of the crown; which he at length determined by observing, that if a solid be immersed in a fluid, it displaces a quantity of that fluid *equal* to its own bulk.

It is recorded that he was so pleased by the discovery of this simple and universal principle, that, springing from the bath in which he was, he rushed naked out of the house, exclaiming εὕρηκα, "I have found it."

Unfortunately, however, in cases where two substances enter into a chemical combination, the solution given above is not always accurate; for two substances when chemically combined do not necessarily occupy the same space that they did when separate. Thus a pint of water, and a pint of sulphuric acid, when mixed together, do not make a quart. The density of bell metal, which is a mixture of copper and tin, is greater than the density of either of the two metals which compose it.

Although, therefore, the solution given above would be perfectly accurate, if the gold and silver were merely joined together mechanically; it could not be relied upon if the metals were melted together, and chemically united, as was in all probability the case.

15. A prism of wood floats in a vessel of water, its specific gravity being .8. What portion of the prism will be above the surface?

16. A cylinder floats in a fluid whose specific gravity is .9, with $\frac{1}{5}$ of its bulk above the surface. What is the specific gravity of the cylinder?

17. A cylinder whose specific gravity is .8 floats in a fluid with $\frac{2}{3}$ of its bulk below the surface. What is the specific gravity of the fluid?

18. A body floats in a fluid with 3 cubic inches above, and 8 cubic inches below the surface. Compare the specific gravities of the body and the fluid.

19. A lump of silver which weighs 130lbs. loses 13lbs. when weighed in water. Find the specific gravity of silver.

20. A lump of metal weighs 40lbs. in air, and 36 in water. Find its specific gravity.

21. A lump of platinum weighs 180 grains in air, and 164 grains in a fluid whose specific gravity is 1.92. Find the specific gravity of platinum.

22. A piece of marble whose specific gravity is 2.7 weighs 60oz. in air, and 40oz. in a fluid. Find the specific gravity of the fluid.

23. A lump of glass weighs 50 grains in air and 34 in alcohol. If the specific gravity of glass be 2.5, what is the specific gravity of alcohol?

24. A block of wood weighs 4lbs. in air; a lump of lead which weighs 11lbs. in air and 10lbs. in water being added to it as a sinker, the two together are found to weigh 9lbs. in water. Find the specific gravity of the wood.

25. A piece of cork weighs 150 grains in air; a piece of metal which weighs 420 grains in alcohol is attached to it, and it is found that the two together weigh 70 grains in alcohol. Find the specific gravity of cork, that of alcohol being .8.

26. To a piece of metal which weighs 6lbs. in air, a piece of platinum which weighs 5lbs. in mercury is attached as a sinker, and the two together are found to weigh 2lbs. in mercury. Find the specific gravity of the metal, that of mercury being 13.5.

27. An hydrometer floats in two fluids with 80 and 70 divisions of the stem above the surface. Each division of the stem contains $\frac{1}{2000}$ of the whole bulk of the hydrometer. Compare the specific gravities of the fluids.

28. The same hydrometer floats in water with 43 divisions above the surface, in milk with 100. Find the specific gravity of milk.

29. The same hydrometer floats in rectified alcohol with 20 divisions above the surface, in common alcohol with 80. Find the specific gravity of common alcohol, that of rectified alcohol being .8.

30. A vessel containing water is placed in one scale of a balance, and counterpoised by a weight placed in the other scale. If a cubic inch of lead be suspended in the water, so that none of the water is spilt, what additional weight must be placed in the other scale to restore equilibrium?

31. Which weighs the most in vacuo, a pound of lead, or a pound of feathers?

32. How many cubic inches of cork must be added to 10 cubic inches of manganese to make the two together of the same specific gravity as water, the specific gravities of cork and manganese being .24 and 8?

33. Two fluids which do not mix meet at the bottom of a bent tube, their heights are as 4 : 5, and the specific gravity of the lighter fluid is .8. Find the specific gravity of the other.

34. Two bodies of different bulks weigh the same in water, which will weigh the most in mercury?

35. In example 30, if there were placed on the top of the water, a piece of wood whose specific gravity was such that it floated with two cubic inches below the surface, what additional weight would be required in the other scale?

36. A piece of wood floats in two fluids which do not mix, so that $\frac{2}{3}$ of its bulk is in the upper fluid; if the specific gravities of the fluids are .8 and .9, what is the specific gravity of the wood?

37. A body floats in two fluids which do not mix, and whose specific gravities are .8 and 1.2. The two surfaces of the upper fluid divide the body into three parts, which are to each other as 4, 3 and 3, the upper part being the largest of the three. Find the specific gravity of the body.

PROPS. 12, 13, 14. It might, at first sight, be doubted whether the air, in which we live and move, inasmuch as under ordinary circumstances it has no quality that is palpable to the senses, be a material substance or not.

But we learn from the researches of philosophers, that, though noiseless when at rest, it is the medium of sound; though inodorous when pure, it is the medium of scent; that to its refraction we owe the phenomenon of twilight, to its motion that of wind, that it is its pressure which gives stability to bodies, enables the fly to crawl upon the ceiling, and fastens the limpet to its native rock, that without it there would be neither clouds nor showers, neither springs nor rivers, that it is the supporter of combustion, that upon its supply and purity depend the health and life of animals and vegetables; that when it is vitiated they sicken, when it is withheld, they die. Such are the purposes which it serves in the operations of nature; nor are the uses to which it is applied in the arts of life either few or unimportant. It is the force exerted by the air, when in motion, that turns the sails of a windmill, and propels a ship through the water; whilst on its pressure and elasticity depend various instruments of the most general utility, such as the air-pump, the condenser, the siphon, the barometer, some forms of the steam engine, and most of the varieties of pumps.

We may satisfy ourselves that the air offers a resistance to bodies in motion by moving an open umbrella slowly against it; this is the principle upon which are constructed those very useless and dangerous contrivances which are called parachutes; and it is owing to this resistance that different bodies let fall from the same height do not reach

the ground in the same time. If a guinea and a feather be let fall in an exhausted receiver, they will be found to fall equally fast. But the consideration of the effects due to the resistance of the air does not come within the scope of the present treatise, inasmuch as the instruments enumerated in the schedule depend only upon its pressure and elasticity.

The pressure of the air may be illustrated by many and very simple experiments.

1. If a glass tumbler be filled with water, and a piece of paper be pressed closely against its mouth, the tumbler may be inverted without the water escaping. This proves that the pressure of the air upon the paper, is greater than the weight of the water in the glass. If a hole could be made in the upper part of the glass, the air would press the water downwards with the same force as that with which it presses it upwards and the water would fall by its own weight. The same thing would occur if the paper were away, for it would be impossible to hold the glass perfectly level or perfectly still, and therefore the equilibrium, which is unstable, cannot possibly subsist in practice.

2. Again, if a glass tube open at both ends be placed upright on the plate of an air-pump, and the hand be laid on the top of the glass so as to cover it; then if the air be exhausted from the glass, the external air will press the hand against the glass so that it is scarcely possible to move it; upon letting in the air the hand will be loosened again.

3. If the mouth piece of a pair of bellows be stopped so that the air cannot enter, a very great effort will be required to separate the boards.

The atmosphere, therefore, exerts a pressure upon all things with which it is in contact, and this pressure is subject to the same laws as the pressures exerted by other fluid bodies. It will be the same, therefore, at all points in the same level, and will increase with the depth. The pressure of the air therefore, is not the same at different heights, but if the difference of the heights be only a few inches or a few feet, the difference of the atmospheric pressures will be so small that in practice they may be considered as equal.

4. Flies and some other insects are able to create a vacuum between their feet and the surface on which they tread, and it is the pressure of the air upon the upper part of their feet, which enables them to walk upon the window or the ceiling. On the same principle may be explained the adhesion of shell-fish to rocks.

5. We have already stated that the air exerts a pressure of about 15 lbs. on the square inch, now the surface of a man's body contains about 2000 square inches, and the whole pressure of the air upon his body will amount to 30000 lbs. This enormous pressure upon our bodies occasions us no annoyance; because, in the first place, the air presses equally in all directions, so that no part of our body is displaced; and, secondly, the pressure of the external air is opposed by the equal and contrary pressure which is exerted by the elastic fluids contained within the body.

6. If one end of an open tube be placed beneath the surface of a vessel of water, it is possible to drink the water (as indeed every schoolboy knows) by the operation which is vulgarly called suction. In this operation, the air is exhausted from the tube by drawing in the breath, but it is the pressure of the external air that forces the water up the tube.

A great number of experiments illustrating the atmospheric pressure may be found in Desagulier's Course of

Experimental Philosophy, a work remarkable not only for the number of experiments recorded in it, but also for the clearness with which they are described.

We proceed to enumerate a few phenomena which illustrate the elasticity of the air.

7. If a shrivelled apple be placed under the receiver of an air-pump, when the air is exhausted from the receiver, the air inside the apple will expand, and distend the skin of the apple so as to make it appear as smooth as one fresh gathered. If the air be readmitted into the receiver, the apple will return to its former shape.

8. If a thin glass bottle full of atmospheric air be hermetically sealed*, and placed under the receiver of an air-pump, when the air is exhausted from the receiver, the elastic force of the enclosed air will burst the bottle.

9. If a vessel filled with beer or luke-warm water be placed under the receiver, when the air is exhausted from the receiver, the air in the liquid will expand and rise to the top in large bubbles. In the case of luke-warm water, these bubbles will be so large that the water will appear almost to boil.

10. When a cupping glass is applied to the body, the air within the cupping glass is rarified, (either by means of an air-pump, or, as is more frequently the case, by burning a torch within the glass,) and the pressure of the external air upon the skin being thus diminished, the pressure of the fluids within extends and swells out the flesh ready for the lancets.

11. If a piece of lead be attached to a piece of cork so that the two together will just sink in water; then if the two be immersed in a vessel full of water, and the vessel

* A vessel is said to be hermetically sealed, when it is made air-tight, so that no air can enter into it, or escape from it.

placed under the receiver of an air-pump; as the air is exhausted from the receiver, the air in the pores of the cork will expand, and swell the cork, which therefore will become specifically lighter than before, and will rise to the surface. If the air be re-admitted into the receiver, the cork will sink to the bottom again.

12. Fishes, whose bodies are a little heavier than water, have bladders in their bodies filled with air, which they can expand or contract at pleasure. By expanding the air-bladder they become specifically lighter than the water, and are thus enabled to rise to the surface; again when they contract the air-bladder they make themselves specifically heavier than the water, and sink. If the bladder breaks, the fish remains at the bottom, as is the case also with flat fish, such as soles, which have no air bladder.

13. If a vessel of water containing live fish be placed under the receiver of an air-pump, when the air is exhausted from the receiver, the air contained in the air bladders of the fish will expand, and the fish, becoming specifically lighter than water, will rise to the surface. If the air be re-admitted, they will swim below the surface as before.

14. In the well known toy represented by the annexed woodcut; the glass figure inside the jar is hollow, with small holes in the feet. A quantity of air is contained within the figure, so that its specific gravity is a little less than that of water, and it will therefore float upon the surface. If the finger be pressed upon the covering, the air which lies between the water and the covering will be slightly compressed, and this pressure will be transmitted by the water to the air within the figure; this portion of air, therefore, will be slightly condensed, and a small additional quantity of water will be admitted into the figure, so that its specific gravity will become greater than that of the water, and it will sink.

On removing the finger, the air in the jar will expand again, and the pressure being thus removed from the air in the figure, it will expand likewise, and the figure will become specifically lighter than water as before, and will therefore rise to the surface again.

Prop. 15. It is impossible to exhaust the air from a receiver completely, even by the best air-pump, for after a certain degree of exhaustion has been produced, the elastic force of the air in the receiver will not be sufficient to open the barrel valve, and no farther rarefaction can be produced.

Besides this, the piston cannot be made to fit the barrel accurately; there will therefore always be a certain space below the piston valve, when the piston is at the bottom of the barrel. Now, in order that the piston valve may open, the elastic force of the air contained in this space must be greater than the atmospheric pressure. As soon, therefore, as the rarefaction has proceeded so far, that the air contained in the barrel can be condensed into this space without its elastic force becoming greater than the atmospheric pressure, the piston-valve will not open, and the pump will no longer work.

It is by means of the air-pump, that almost all the properties of the air have been discovered. The air-pump is generally furnished with two barrels, the pistons of which are worked up and down by a rack wheel, which is turned by a winch, and is constructed so that as one piston ascends, the other descends.

Prop. 16. There is no limit to the degree of condensation that may be effected by means of this instrument, provided the instrument can be made sufficiently strong to bear the pressure of the condensed air, and a sufficient force can be applied to the piston.

The reader will see that this instrument differs from

the air-pump in its construction only in the following particular, that its valves open inwards instead of outwards.

The condensation of air is applied to many useful purposes, amongst which may be enumerated the blowing cylinders of furnaces, and air-guns.

A child's pop-gun acts upon the same principle.

PROP. 17. The air decreases rapidly in density, as we ascend above the surface of the earth, and it has been calculated that it does not extend more than fifty miles above the surface.

As we ascend, therefore, to greater heights, the atmospheric pressure will be lessened, and the altitude at which the mercury stands in the barometer will be lessened also, or in common language, the barometer will fall.

It is obvious, therefore, that the barometer may be used for the purpose of measuring heights. At the level of the sea, the mercury stands at about 30 inches, and falls about an inch for every 1000 feet that we ascend.

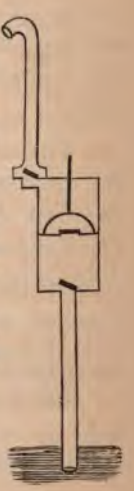
In order, however, to determine accurately the elevation of any place above the level of the sea, we must have recourse to a series of very careful experiments, and somewhat abstruse calculations.

One form of the barometer is also in very general use as a weather-glass; but the words fair, change, &c. commonly engraved on these instruments are entitled to little attention. In general, however, the rising of the mercury indicates the approach of fair weather; and foul weather generally preceded by a fall of the barometer.

The atmospheric pressure, therefore, is less in foul weather than in fair, and the oppression which we feel at such times, and which we incorrectly ascribe to the heaviness of the air, is in reality owing to the diminution of the atmospheric pressure.

Water barometers may sometimes be found in extensive collections of philosophical instruments, but their great length renders them very inconvenient; and mercurial barometers, whose length is only about three feet, have come into general use. The tube of a water barometer must be nearly forty feet in length.

PROPS. 19, 20. The water can only be raised by the atmospheric pressure, to a height of about 30 feet, but it may be lifted by the ascent of the piston to a much greater height. If a conduct pipe with a valve at the bottom opening upwards, be joined to the spout of a common pump, a quantity of water will be lifted up into this pipe at each ascent of the piston; when the piston descends the pressure of the water in this pipe will close the valve, and prevent the return of the water into the barrel. A pump fitted with this contrivance is called a lifting pump.



The machine used in public houses for drawing beer is an application of the lifting pump; the common fire engine is a double forcing pump.

THE AIR VESSEL.

In the forms of pumps which we have described, the stream of water is intermittent; thus in the lifting pump the stream will flow only during the ascent, and in the forcing pump during the descent of the piston. But the stream may be made continuous by the addition of a contrivance called the air vessel, or air spring.

This consists of a close vessel filled with air, which is connected with the pipe through which the water is discharged. The water being forced into this vessel, the air in the upper part of it becomes condensed, and constantly

exerts a pressure upon the surface of the water. The orifice of the discharge pipe communicates with the lower part of the vessel, and therefore this constant pressure will force the water through the pipe in a continued stream.

EXAMPLES.

1. To find the quantity of air left in the receiver of an air-pump after any number of strokes of the piston, the content of the receiver being 10 times that of the barrel.

The air contained in the barrel will be one eleventh of the air contained in the barrel and receiver together. At the first descent of the piston this eleventh part will be expelled, and ten elevenths will remain.

Of the air that remains, one eleventh will again be expelled at the second descent of the piston, and ten elevenths will remain.

Calculating in this way we shall have

after one stroke, $\frac{1}{11}$ pumped out, $\frac{10}{11}$ remaining,

after two strokes, $\frac{21}{121}$ pumped out, $\frac{100}{121}$ remaining,

after three strokes, $\frac{331}{1331}$ pumped out, $\frac{1000}{1331}$ remaining,

and so on.

2. In the former example, what will be the density of the air, and its pressure, on a square inch after two descents of the piston: the original pressure of the air in the receiver being $14\frac{2}{5}$ lbs. on the square inch?

The quantity of air remaining in the receiver after two strokes of the piston will be $\frac{10}{11} \times \frac{10}{11}$ of the original *quantity.*

Its density therefore will be to the density of the air originally contained in the receiver as $\frac{100}{121} : 1$. And, since the elastic force of air varies as the density, its pressure on the square inch : $14\frac{2}{5} :: \frac{100}{121} : 1$.

Hence its pressure on the square inch $= \frac{72}{5} \times \frac{100}{121}$ lbs. $= \frac{1440}{121}$ lbs.

$= 11\frac{109}{121}$ lbs.

3. To find the quantity of air contained in the receiver of a condenser, after any number of descents of the piston; the content of the receiver being ten times that of the barrel.

It is obvious that, since $\frac{1}{10}$ is pumped in at each stroke,

after one stroke the receiver will contain $\frac{11}{10}$,

two..... $\frac{12}{10}$,

three..... $\frac{13}{10}$,

and so on.

4. In the above example, what will be the density of the air, and its pressure on a square inch after two descents of the piston; the original pressure of the air in the receiver being $14\frac{2}{5}$ lbs. to the square inch?

The density of the air in the receiver after two strokes of the piston, will be $\frac{12}{10}$ of its original density,

and its pressure on the square inch : $14\frac{2}{5}$ :: $\frac{12}{10}$: 1 ;

$\therefore$ its pressure on the square inch $= \frac{72}{5} \times \frac{12}{10} = \frac{864}{50}$,

$= 17\frac{14}{50}$lbs.

5. If the weight of a cubic inch of mercury be $7\frac{4}{5}$ oz., what is the pressure of the air on a square inch when the mercury stands at $29\frac{1}{2}$ inches?

The pressure of the air on a square inch = the weight of a column of mercury whose base is a square inch, and height $29\frac{1}{2}$ inches,

= weight of $29\frac{1}{2}$ cubic inches of mercury,

$= 29\frac{1}{2} \times 7\frac{4}{5} = \frac{59}{2} \times \frac{39}{5}$oz. $= \frac{2301}{10}$oz.

= 230.1 oz. = 14.38125lbs.

6. If the mercurial barometer stands at 32 inches, at what height will a water barometer stand, the density of mercury being 13.5?

Let x = the height of the water barometer.

32×13.5 = weight of a column of mercury whose base is one square inch, and height 32 inches,

= atmospheric pressure on the square inch,

= weight of a column of water whose base is a square inch, and height is $x = x \times 1$;

$\therefore$ $x = 32 \times 13.5$ inches = 432 inches,

= 36 feet.

7. If the content of the receiver of an air-pump be 3 times that of the barrel, what will be the pressure on a square inch of the internal surface of the receiver after three strokes of the piston, the atmospheric pressure being 15 lbs. on a square inch?

8. If the content of the receiver of a condenser be 12 times that of the barrel, after how many strokes will the elastic force of the air in the receiver be exactly double the atmospheric pressure?

9. If a cubic foot of water weigh 1000 oz., at what height will the water barometer stand, when the atmospheric pressure is 15 lbs. on a square inch?

10. If a cubic inch of mercury weighs 8 oz., and the barometer stands at 30 inches, what will be the atmospheric pressure on a square inch?

11. If, when the barometer stands at 28 inches, the atmospheric pressure is 14 lbs. on a square inch, what will be the atmospheric pressure on a square inch when the barometer stands at 30 inches?

12. The content of the receiver of an air-pump is 5 times that of the barrel; a barometer being introduced into the receiver, is observed after one ascent of the piston to stand at 27 inches, at what height will it stand after two more ascents of the piston?

13. The content of the receiver of a condenser is 10 times that of the barrel. A barometer being placed in it is observed to stand at 30 inches. After how many descents of the piston would the barometer stand at 33 inches?

14. Explain the construction and operation of the common syringe.

15. Explain the construction and operation of the common bellows.

16. If it were required to raise mercury by means of a pump, how long might the suction pipe be?

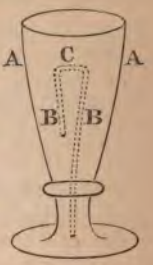
PROP. 21. It is evident from the proof here given, that if the leg in contact with the liquid be the longer of the two, the liquid will flow back into the vessel, as soon as the other end is unstopped. If the legs are of the same length the fluid will remain suspended.

Instead of exhausting the siphon, which is sometimes a difficult process, it may be inverted and filled with water; then if the shorter leg be placed beneath the surface of the liquid, and both ends unstopped, the fluid will begin to flow from the longer leg.

It is evident from what we have stated above, that the siphon can only be used for transferring liquids from a higher to a lower level; when, therefore, it is used for transferring a liquid from one vessel into another, it will cease to act as soon as the levels of the liquid in the two vessels are the same. Siphons are constantly used in chemical experiments where it is necessary to draw off a liquid without disturbing the sediment; they are used also wherever it is necessary to draw off liquids from the upper part of vessels without making an orifice below.

The height of the shorter leg of the siphon must always be less than the height of the water-barometer; for, if not, the atmospheric pressure will be unable to support the weight of the fluid in the shorter leg, and the continuity will be interrupted. If, however, a fluid heavier than water be used, the height of the column of fluid, which the atmospheric pressure will just support, will be diminished; and therefore the extreme height of the shorter leg will be diminished also. Thus, if the siphon be used for decanting mercury from one vessel to another, the height of the *shorter leg* must not be greater than 30 inches.

The action of intermittent springs may be illustrated by the following toy, which the reader can easily make for himself, if he cannot procure it at a common toy shop.



The shorter leg of the siphon *B* fits into the vessel *A*. If the vessel be placed so that water may run into it from a tube *D*,* as soon as the water rises in the vessel to the top *C* of the siphon, it will begin to act, and the water will flow out through the longer leg.

If, therefore, the bore of the siphon be such that more water flows out of the vessel by the siphon, than comes into it through the tube, the siphon will gradually empty the vessel again, until the level of the water falls below the mouth of the shorter leg. When this occurs the siphon will cease to act, and its action will not be renewed until the water again rises in the vessel to the point *C*.

Although, therefore, the stream through *D* may be constant, the flow of the water through *B* will be intermittent. We may vary the intervals of rest, by varying the bore of the tube *D* and the siphon *B*.

If we suppose the annexed figure to represent a cavern in the interior of a mountain, into which water flows by a small feeder, and from which it discharges itself by a bent channel or natural siphon, the siphon will not begin to act until the water has risen so as to fill it; and if the siphon discharges the water from the cavern faster than the feeder supplies it, the water will again sink below the end of the siphon, and it will cease to act. Such a spring, therefore, would be intermittent; and the period, during which the spring

* The tube *D* has been omitted in the figure.

would cease running would depend on the comparative size of the feeder and the siphon.

On the same principle are constructed the toys called Tantalus cups. These contain a siphon which is concealed by some contrivance, such as the figure of a man fixed in the middle of the cup. If the cup be filled to a certain point, it will hold the water, but if it be either filled or tilted so that the water rises above this point, the siphon in the figure begins to act, and discharges the water into a receptacle in the bottom of the cup. It is impossible, therefore, to fill such a cup to the brim, or to drink from it.

Props. 22, 23. It is obvious that any scale of graduation would serve equally well to measure the temperature, but the centigrade and Fahrenheit's scales are the most convenient, and therefore the most generally used.

Any substance that expands by heat and contracts by cold would serve to measure degrees of temperature; and, accordingly, thermometers have been constructed of different substances, such as air, metals, spirits of wine, linseed oil, &c. But mercury answers better on the whole than any other liquid, because its expansion is very uniform, and it can endure considerable degrees of heat and cold without boiling or freezing.

The freezing point is the same under all circumstances. The boiling point varies according to the pressure of the atmosphere, but is always the same under the same atmospheric pressure. The height at which the thermometer stands when the height of the barometer is 30 inches, is used as the standard for the boiling point.

It is assumed that, between the freezing and boiling points, mercury expands equally with equal additions of heat. Although this is not exactly the case, it has been calculated that the unequal expansion of the mercury in a thermometer is compensated by the expansion of the glass, so that we may on the whole consider the mercurial thermometer as *an accurate* indicator of the changes of temperature.

EXAMPLES.

1. Water is flowing out of a vessel through a siphon. What would take place if the pressure of the atmosphere were removed and afterwards restored; (1) when the lower end of the siphon is immersed in water, (2) when it is not?

In both cases, if the atmospheric pressure be removed, the fluid will fall down in both legs, leaving a vacuum in the tube.

(1) When the lower end of the siphon is immersed in water; as soon as the atmospheric pressure is restored, it will force the fluid up both legs, until the two columns of fluid meet, when the action of the siphon will be restored.

But (2) When the lower end of the siphon is not immersed in water; as soon as the atmospheric pressure is restored, it will exert the same force on the surface of the water in the tube that it does on the water outside the tube, therefore the water will not rise in the tube, and the action of the siphon will not be restored.

2. What point of the centigrade scale corresponds to 95^0 of Fahrenheit's scale?

Let C^0 be the number of degrees in the centigrade scale.

$$C^0 : 95^0 - 32^0 :: 100^0 : 180^0,$$

$$C^0 = \frac{63 \times 100}{180} = \frac{63 \times 5}{9} = 35^0.$$

3. What point of the Fahrenheit scale corresponds to 80^0 of the centigrade scale?

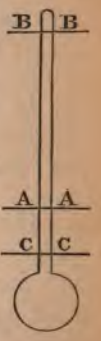
Let F^0 be the number of degrees in the Fahrenheit scale.

$$F^0 - 32^0 : 80^0 :: 180^0 : 100^0,$$

$$F^0 - 32^0 = \frac{80 \times 180}{100} = 144^0;$$

$$\therefore\ F^0 = 176^0.$$

4. What degree of the centigrade scale corresponds to 17^0 Fahrenheit?



If C be the point at which the mercury stands,

number of degrees Fahrenheit in AC : number of degrees Fahrenheit in AB :: number of degrees centigrade in AC : number of degrees centigrade in AB,

$$F^0 \text{ in } AC : 180 :: C^0 \text{ in } AC : 100,$$

$$C^0 \text{ in } AC = \frac{100}{180} \times F^0 \text{ in } AC = \frac{5}{9}(32 - 17)$$

$$= \frac{5}{9} . 15 = \frac{25}{3} = 8\tfrac{1}{3};$$

and since C is below freezing point, which is the zero of the centigrade scale, this must be reckoned as negative.

Therefore 17^0 corresponds to $-8\frac{1}{3}^0$ on the centigrade scale.

5. What degree of the Fahrenheit scale corresponds to -25^0 of the centigrade?

As in the former example,

$$F^0 \text{ in } AC : 25^0 :: 180 : 100;$$

$$\therefore\ F^0 \text{ in } AC = \frac{25 \times 180}{100} = 45^0.$$

Therefore C is 45^0 below freezing point, and will correspond to $32^0 - 45^0$, or -13^0 of the Fahrenheit scale.

6. What degree of the Fahrenheit scale corresponds to

(α) 65^0,

(β) 30^0,

(γ) $8\frac{1}{3}^0$,

(δ) -5^0,

and (ϵ) -25^0, of the centigrade?

7. What degree of the centigrade scale corresponds to

(α) 185^0, (β) 68^0, (γ) 41^0,

(δ) 14^0, (ϵ) 23^0, (ζ) -4^0, (η) -13^0,

and (θ) -22^0 of the Fahrenheit?

STEAM ENGINE.

1. If water be heated up to the temperature of 212^0 Fahrenheit, it will be gradually converted into an elastic fluid, called steam. This fluid is not permanently elastic, like air, but may be reconverted into water by diminishing its temperature. A cubic inch of water will form about 1700 cubic inches of steam; it follows, therefore, that if 1700 cubic inches of steam be enclosed in a vessel, and the steam be *condensed* (i. e. reconverted into water) by the application of cold, it will, after condensation, occupy only the space of one cubic inch, leaving a vacuum in the rest of the vessel. The elastic force and condensation of steam are employed in various ways so as to move an air-tight piston up and down in a cylinder.

(1) In the *atmospheric engine*, steam being admitted into the lower part of the cylinder, the piston is raised by means of a counterpoising weight; the steam in the cylinder is then condensed by the application of cold water, so as to form a vacuum within the cylinder; when the piston is forced down again by the atmospheric pressure.

(2) The *single-acting low-pressure* engine differs from the atmospheric chiefly in this respect, that the piston is

not forced down by atmospheric pressure, but by the elastic force of steam admitted into the upper part of the cylinder.

(3) In the *double-acting low-pressure* engine, a vacuum is formed alternately above and below the piston, which is forced up as well as down by the pressure of steam on the other side. Hence, a counterpoising weight is unnecessary.

All these forms of the steam engine depend upon the condensation of steam, and are therefore called *condensing engines.*

(4) But the piston may be moved up and down by the elastic force of steam, without any condensation at all, provided the steam be allowed to escape at the end of each stroke. Steam engines of this kind are called *noncondensing engines;* they are also called *high-pressure engines,* for the following reason. In these engines the motion of the piston is opposed by the atmospheric pressure, the elastic force, therefore, of the steam must be sufficient to overcome the atmospheric pressure as well as to raise the piston. It is necessary, therefore, to use steam of a very high temperature and pressure in these engines, for the pressure of steam at 212^0 is very nearly the same as that of the atmosphere.

The motion of the piston being thus produced, it may be applied to any mechanical purpose by means of various contrivances. It does not come within the scope of the present treatise to enter into any details concerning the contrivances by which it is so applied; on this subject the reader may consult the treatise on the steam engine by Dr Lardner.

CAPILLARY ATTRACTION.

Cohesion. The attraction which the particles of a body exert upon each other is called cohesion. The force of cohesion is found to be different in different substances, it is very strong in iron, less strong in wood, and weaker still *in ice.* According to our definition, a fluid is a substance

whose parts may be moved among each other by any force however small, the particles of a fluid, therefore, can possess no cohesion, and as water and other liquids possess some slight degree of cohesion, it follows that they do not possess perfect fluidity. Their cohesion, however, is so exceedingly small, that the conclusions founded on the definition may be considered as practically correct.

The existence of cohesion in liquids is proved by the spherical form which a drop of water or mercury will assume, and by the tendency which it has to resume that form when slightly pressed.

Adhesion. Cohesion takes place not only between the particles of the same body; but also between the particles of different bodies, when brought into contact; this variety of attraction has been called adhesion, although some writers confine this term to the mutual attraction which takes place between solids and liquids.

When a solid is in contact with a liquid it will be wetted, if the force of adhesion is stronger than the cohesive force of the liquid; but if the latter force exceeds the former it will remain dry. Thus, if one's finger be immersed in water it becomes wet, if it be immersed in quicksilver it remains dry. Quicksilver poured into gauze will not run through, nor will a tallow candle be wetted if dipped in water. Even those fluids which do not moisten the solids immersed in them possess a slight degree of adhesion to them; thus small drops of mercury continue hanging to glass and other bodies which they do not wet, even when held in such a position that they would naturally fall. By means of this principle may be explained the fact, that if two bodies floating in a liquid be placed at small distances from each other they will approach each other when both or neither of them are wetted by the liquid, and recede from each other when one is wetted and the other is not wetted. A similar attraction or repulsion will take place between a floating body and the sides of the vessel in which the liquid is contained.

Capillary attraction. To the adhesion which exists between solids and liquids is attributed the phenomenon of capillary attraction. When water is contained in a glass, it will be found, if we observe the edge of the water, that it is not quite level close to the glass, but is somewhat raised, so as to stand a little higher at the edge than it does in the middle; this elevation, which may be observed most readily when the glass is filled to the brim, is generally supposed to be caused by the attraction of the sides of the glass, although some doubt still remains respecting the explanation of the fact. In the same manner, if a plate of glass be immersed in water, the water will rise when it is in contact with the plate; but this elevation is most perceptible in tubes of small bore; and the smaller the tube the higher the water will rise. Tubes of this very small bore are called capillary, from *capillus* a hair, and hence this species of attraction is called capillary attraction. This elevation is not confined to water and glass, but occurs in all solids and liquids where the solid is capable of being wetted by the liquid. If the solid is incapable of being wetted by the liquid the circumstances are reversed, and the liquid will sink in a ring round the solid.

Many natural phenomena are attributable to capillary attraction, as the absorption of liquids by sponge or blotting paper; the ascent of oil or melted tallow in the wicks of lamps and candles, of water in a lump of sugar, or in a wall with a damp foundation; and the swelling of wood when wetted.

Capillary siphon. If one end of a hank of cotton be immersed in a vessel containing water, the water will gradually pervade the whole of the cotton; if the other end of the hank be lower than the surface of the water in the vessel, the cotton will act as a siphon and will gradually empty the vessel of its water. In the same manner we may empty a basin of water, by hanging over the edge a towel *which dips in the* water.

ANSWERS TO THE EXAMPLES.

N. B.—The Specific gravities given in the following examples are not always accurate, being in some instances merely loose approximations to the exact values.

INTRODUCTORY CHAPTER.

8. 15 : 16.
9. Vid. notes.
10. Vid. notes.
11. 4450 : 3833.
12. 8.
13. 4 : 3.
14. 7 . 8725.
15. 7.
16. 7 cubic inches.
17. 12 cubic feet.
18. 15 lbs.
19. $4\frac{4}{5}$ lbs.
21. 11.
22. .866.
23. 1.84.

THE LEVER.

9. 8 lbs.
10. 3 feet.
11. 5 lbs. and 6 lbs.
12. 55 lbs. and 66 lbs.
13. 8 feet and 4 feet.
14. 12 feet and 6 feet.
15. $37\frac{1}{2}$ lbs. and $7\frac{1}{2}$ lbs.
16. 6 feet.
17. 6 lbs. and 2 lbs.
18. 2 feet and 3 feet.
19. The distance of the weight from the stronger man, must be equal to one-third of the length of the pole.
20. 36 lbs.
21. 6 lbs.
22. 80 lbs.
23. 2 inches from the fulcrum.

COMPOSITION AND RESOLUTION OF FORCES.

5. A force of 15 lbs. acting in the same direction.
6. A force of 1 lb. acting in the direction of the greater force.
7. Vid. notes.
8. Vid. notes.
9. 15 lbs.
10. Vid. Ex. 2.
11. $\frac{3}{4}$ of a lb.
12. $20\sqrt{2}$ or 28.284.

MACHINES.

12. 120 lbs.
13. 48 lbs.
14. 1 foot.
15. 1 qr.
16. 1 qr.
17. 3.
18. 384 lbs.
19. 7 lbs.
20. 6.
21. 10 inches and 4 lbs.
22. 64 lbs.
23. 7 qrs.
24. $\frac{3}{4}$ of a qr.
25. 1650 lbs.
26. $3\frac{1}{2}$ feet.
27. 1152 : 5.

VELOCITIES.

6. 3 : 4.
7. 3 : 2.
8. 1 : 6.
9. 12 feet.
10. 5 : 3.
11. 108 yards.
12. 2 : 1.
13. 4 : 5.
14. 4 : 1.
15. 48 yards.
16. 7 : 1.
17. 16 : 1.
18. 12, and one end of the string is fixed to the lower block.
19. 2 : 1.
20. 7 inches in a second.
21. 3 : 1.

CENTER OF GRAVITY.

11. Join the points of bisection of the opposite sides. Vid. Prop. 21.
12. If the two pounds are suspended at the point C of the ΔABC; bisect AB in D; join DC, and bisect it in G. G is the center of gravity.
18. Vid. Ex. 8.
26. $7\frac{1}{5}$ lbs.
29. 80 feet.

For the rest of the examples vid. the notes.

HYDROSTATICS.

N. B.—In the following examples, the experiments are always supposed to take place in vacuo, unless the contrary be expressly stated.

PRESSURE OF FLUIDS.

9. 1 : 16.
10. 12 lbs.
11. 432 lbs.
12. 1 : 2.
13. $52\frac{1}{12}$ oz., and 1875 lbs.
14. Vid. notes.
15. At least one foot.
16. 125 lbs., and 4445 lbs. Vid. notes.
17. Vid. notes.
18. Vid. notes.
19. 6 : 5.

BODIES IMMERSED IN FLUIDS.

15. One-fifth.
16. .72.
17. 1 . 2.
18. 8 : 11.
19. 10.
20. 10.
21. 21 . 6.
22. .9.
23. .8.
24. .8.
25. .24.
26. 9.
27. 193 : 192.
28. 1 . 03
29. .825.
30. Vid. notes.
31. Vid. Ex. 13.
32. $92\frac{2}{19}$ oz.
33. 1.
34. Vid. Ex. 13.
35. Vid. notes.
36. .833.
37. .6.

PUMPS AND BAROMETER.

7. 6 lbs. $\frac{5}{7}$ oz.
8. 12 strokes.
9. 34 . 56 feet.
10. 15 lbs.
11. 15 lbs.
12. 18$\frac{8}{9}$ inches.
13. (See descent.
14. Vid. Prop. 19.
15. Vid. Prop. 16.
16. Vid. notes.

THERMOMETER.

6. α. 149°.
   β. 86°.
   γ. 47°.
   δ. 23°.
   ϵ. −13°.
7. α. 85°.
   β. 20°.
   γ. 5°.
   δ. −10°.
   ϵ. −5°.
   ζ. −20°.
   η. −25°.
   θ. −30.

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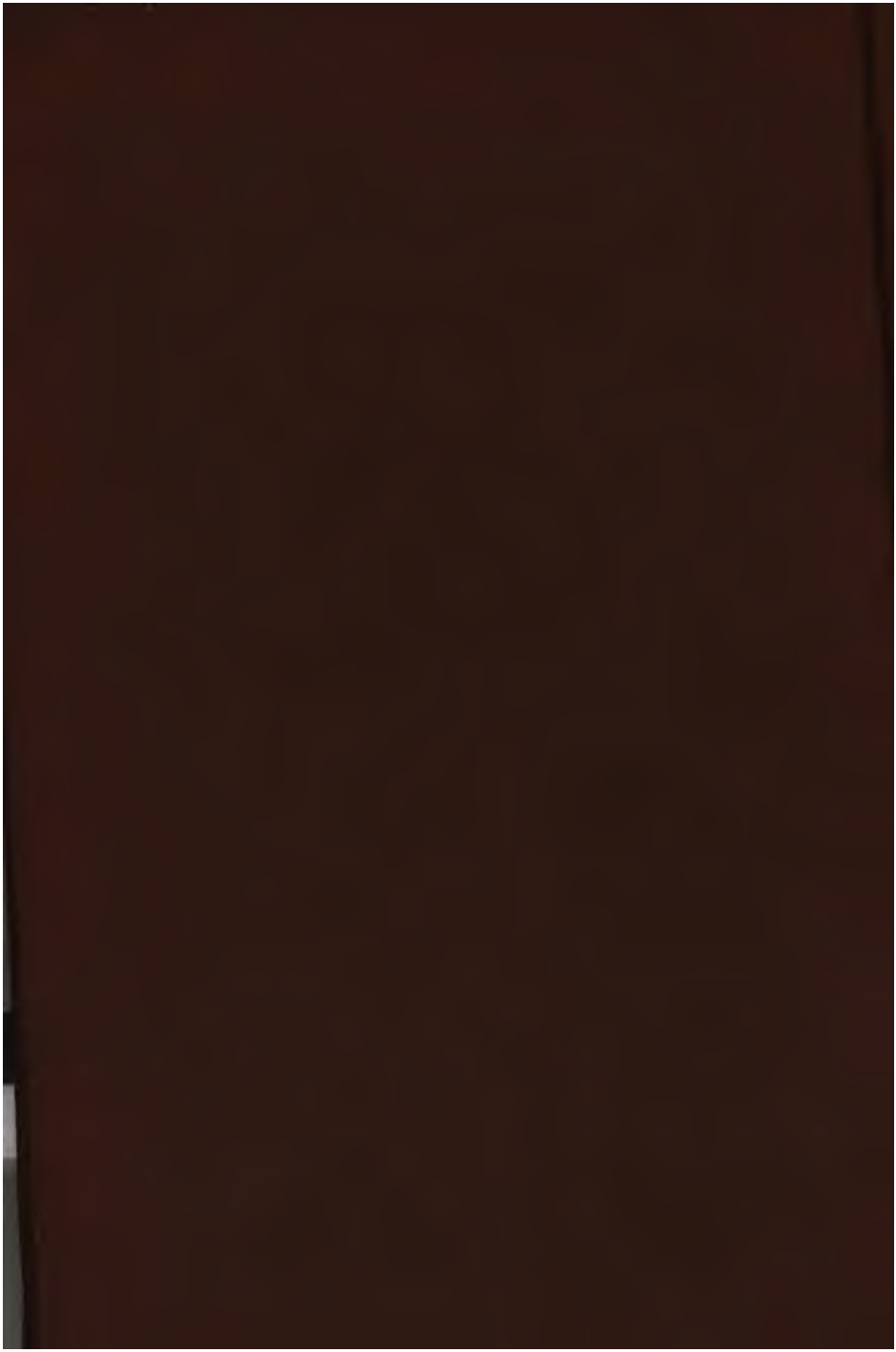
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